Conservation of Energy

- State the law of conservation of energy.
- Describe a closed system.
- Use the law of conservation of energy to solve problems.

There are many energy conversions between potential and kinetic energy as the cars travel around this double looping roller coaster. Throughout the ride, however, there will always be the same total amount of energy.

Conservation of Energy

The law of conservation of energy states that within a closed system, energy can change form, but the total amount of energy is constant. Another way of expressing the law of conservation of energy is to say that energy can neither be created nor destroyed. An important part of using the conservation of energy is selecting the system. Just as in conservation of momentum, energy is conserved only if the system is closed. In a closed system, objects may not enter or leave, and it is isolated from external forces so that no work can be done on the system.

In the analysis of the behavior of an object, you must make sure you have included everything in the system that is involved in the motion. For example, if you are considering a ball that is acted on by gravity, you must include the earth in your system. If considered by itself, one can tell that the kinetic energy of the ball is increasing as it falls, but only by including the earth in the system can you see that the increasing kinetic energy is balanced by an equivalent loss of potential energy. The sum of the kinetic energy and the potential energy of an object is often called the mechanical energy.

Consider a box with a weight of 20.0 N sitting at rest on a shelf that is 2.00 m above the earth. The box has zero kinetic energy but it has potential energy related to its weight and the distance to the earth’s surface.

\[ PE = mgh = (20.0 \text{ N})(2.00 \text{ m}) = 40.0 \text{ J} \]
If the box slides off the shelf, the only force acting on the box is the force of gravity and so the box falls. We can calculate the speed of the box when it strikes the ground by several methods. We can calculate the speed directly using the formula $v_f^2 = 2ad$. We can also find the final velocity by setting the kinetic energy at the bottom of the fall equal to the potential energy at the top, $KE = PE$, thus $\frac{1}{2}mv^2 = mgh$. When reduced, we see that $v_f^2 = 2gh$. Note that these formulas are essentially the same: when gravity is the acceleration and the height is the distance, they are the same equation.

$$v = \sqrt{(2)(9.80 \text{ m/s}^2)(2.00 \text{ m})} = 6.26 \text{ m/s}$$

**Example Problem:** Suppose a cannon is sitting on top of a 50.0 m high hill and a 5.00 kg cannon ball is fired with a velocity of 30.0 m/s at some unknown angle. What is the velocity of the cannon ball when it strikes the earth?

**Solution:** Since the angle at which the cannon ball is fired is unknown, we cannot use the usual equations from projectile motion. However, at the moment the cannon ball is fired, it has a certain $KE$ due to the mass of the ball and its speed and it has a certain $PE$ due to its mass and it height above the earth. Those two quantities of energy can be calculated. When the ball returns to the earth, its $PE$ will be zero. Therefore, its $KE$ at that point must account for the total of its original $KE + PE$. Thus

$$E_{TOTAL} = KE + PE = \frac{1}{2}mv^2 + mgh$$

$$= \left(\frac{1}{2}\right)(5.00 \text{ kg})(30.0 \text{ m/s})^2 + (5.00 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m})$$

$$= 2250 \text{ J} + 2450 \text{ J} = 4700 \text{ J}$$

$$\frac{1}{2}mv_f^2 = 4700 \text{ J}$$

$$v_f = \sqrt{(2)(4700 \text{ J})} = 43.4 \text{ m/s}$$

**Example Problem:** A 2.00 g bullet moving at 705 m/s strikes a 0.250 kg block of wood at rest on a frictionless surface. The bullet sticks in the wood and the combined mass moves slowly down the table.

(a) What is the $KE$ of the bullet before the collision?

(b) What is the speed of the combination after the collision?

(c) How much $KE$ was lost in the collision?

**Solution:**

(a) $KE_{BULLET} = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.00200 \text{ kg})(705 \text{ m/s})^2 = 497 \text{ J}$

(b) $m_Bv_B + m_Wv_W = (m_{B+W})(v_{B+W})$

$(0.00200 \text{ kg})(705 \text{ m/s}) + (0.250 \text{ kg})(0 \text{ m/s}) = (0.252 \text{ kg})(V)$

$(1.41 \text{ kg m/s}) = (0.252 \text{ kg})(V)$

$V = 5.60 \text{ m/s}$

(c) $KE_{COMBINATION} = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.252 \text{ kg})(5.60 \text{ m/s})^2 = 3.95 \text{ J}$

$KE_{LOST} = KE_{BEFORE} - KE_{AFTER} = 497 \text{ J} - 4 \text{ J} = 493 \text{ J}$

**Summary**

- In a closed system, energy may change forms but the total amount of energy is constant.

**Practice**

The following video demonstrates Newton Ball tricks. Use this resource to answer the questions that follow.
1. What happens when one ball is pulled up to one side and released?
2. What happens when three balls are pulled up to one side and released?
3. What happens when two balls are pulled out from each side and released?

Practice problems with answers for the law of conservation of energy:
http://www.physicsclassroom.com/class/energy/u5l2bc.cfm

Review

1. A 15.0 kg chunk of ice falls off the top of an iceberg. If the chunk of ice falls 8.00 m to the surface of the water,
   (a) what is the kinetic energy of the chunk of ice when its hits the water, and
   (b) what is its velocity?
2. An 85.0 kg cart is rolling along a level road at 9.00 m/s. The cart encounters a hill and coasts up the hill.
   (a) Assuming the movement is frictionless, at what vertical height will the cart come to rest?
   (b) Do you need to know the mass of the cart to solve this problem?
3. A circus performer swings down from a platform on a rope tied to the top of a tent in a pendulum-like swing. The performer’s feet touch the ground 9.00 m below where the rope is tied. How fast is the performer moving at the bottom of the arc?
4. A skier starts from rest at the top of a 45.0 m hill, coasts down a 30° slope into a valley, and continues up to the top of a 40.0 m hill. Both hill heights are measured from the valley floor. Assume the skier puts no effort into the motion (always coasting) and there is no friction.
   (a) How fast will the skier be moving on the valley floor?
   (b) How fast will the skier be moving on the top of the 40.0 m hill?
5. A 2.00 kg ball is thrown upward at some unknown angle from the top of a 20.0 m high building. If the initial magnitude of the velocity of the ball is 20.0 m/s, what is the magnitude of the final velocity when it strikes the ground? Ignore air resistance.
6. If a 2.00 kg ball is thrown straight upward with a KE of 500 J, what maximum height will it reach? Neglect air resistance.

- **conservation of energy:** An empirical law of physics (meaning it cannot be derived), states that the total amount of energy within an isolated system is constant. Although energy can be transformed from one form into another, energy cannot be created or destroyed.
- **closed system:** Means it cannot exchange any of heat, work, or matter with the surroundings.
- **mechanical energy:** The sum of potential energy and kinetic energy.
References