Using Algebra in Chemistry

CK12 Editor

AUTHORS
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CONCEPT 1 Using Algebra in Chemistry

Lesson Objectives

The student will:

- perform algebraic manipulations to solve equations.
- use the density equation to solve for the density, mass, or volume when two of the quantities in the equation are known.
- construct conversion factors from equivalent measurements.
- apply the techniques of dimensional analysis to solving problems.
- perform metric conversions using dimensional analysis.

Vocabulary

conversion factor a factor used to convert one unit of measurement into another unit

dimensional analysis a technique that involves the study of the dimensions (units) of physical quantities

Introduction

During your studies of chemistry (and physics as well), you will note that mathematical equations are used in a number of different applications. Many of these equations have a number of different variables that you will need to work with. You should also note that these equations will often require you to use measurements with their units. Algebra skills become very important here!

Solving Algebraic Equations

Chemists use algebraic equations to express relationships between quantities. An example of such an equation is the relationship between the density, mass, and volume of a substance: $D = \frac{m}{V}$ (density is equal to mass divided by volume). Given (or making) measurements of the mass and volume of a substance, you can use this equation to determine the density. Suppose, for example, that you have measured the mass and volume of a sample of liquid mercury and found that 5.00 mL of mercury has a mass of 67.5 grams. Plugging these measurements into the density formula gives you the density of mercury.

$$D = \frac{\text{mass}}{\text{volume}} = \frac{67.5 \text{ g}}{5.00 \text{ mL}} = 13.5 \text{ g/mL}$$

You should notice both units and significant figures are carried through the mathematical operations.

Frequently, you may be asked to use the density equation to solve for a variable other than density. For example, you may be given measurements for density and mass and be asked to determine the volume.

Example:

The density of solid lead is 11.34 g/mL. What volume will 81.0 g of lead occupy?

Since $D = \frac{m}{V}$, then $V = \frac{m}{D}$.

$$V = \frac{81.0 \text{ g}}{11.34 \text{ g/mL}} = 7.14 \text{ mL}$$

A common equation used in chemistry is $PV = nRT$. Even without knowing what these variables represent, you can manipulate this equation to solve for any of the five quantities.

$$P = \frac{nRT}{V} \quad V = \frac{nRT}{P} \quad n = \frac{PV}{RT} \quad R = \frac{PV}{nT} \quad T = \frac{PV}{nR}$$

Make sure you recall these skills from algebra. If necessary, you should practice them.

Example:

Use the equation $\frac{A}{B} = \frac{C}{D}$ and the values $A = 15.1$ g, $B = 3.000$ mL, and $C = 326.96$ grams to determine the value of D.

$$D = \frac{BC}{A} = \frac{(3.000 \text{ mL})(326.96 \text{ g})}{(15.1 \text{ g})} = 65.0 \text{ mL}$$

The calculator-determined value for this arithmetic may yield 64.956954 mL but you now know not to report such a value. Since this answer only allows three significant figures, you must round the answer to 65.0 mL.

Conversion Factors

A **conversion factor** is a factor used to convert one unit of measurement into another unit. A simple conversion factor can be used to convert meters into centimeters, or a more complex one can be used to convert miles per hour into meters per second. Since most calculations require measurements to be in certain units, you will find many uses for conversion factors. What must always be remembered is that a conversion factor has to represent a fact; because the conversion factor is a fact and not a measurement, the numbers in a conversion factor are exact. This fact can either be simple or complex. For instance, you probably already know the fact that 12 eggs equal 1 dozen. A more complex fact is that the speed of light is 1.86×10^5 miles/second. Either one of these can be used as a conversion factor, depending on the type of calculation you might be working with.

Dimensional Analysis

Frequently, it is necessary to convert units measuring the same quantity from one form to another. For example, it may be necessary to convert a length measurement in meters to millimeters. This process is quite simple if you follow a standard procedure called dimensional analysis (also known as unit analysis or the factor-label method). **Dimensional analysis** is a technique that involves the study of the dimensions (units) of physical quantities. It is a convenient way to check mathematical equations. Dimensional analysis involves considering the units you presently have and the units you wish to end up with, as well as designing conversion factors that will cancel units you don't want and produce units you do want. The conversion factors are created from the equivalency relationships between the units. For example, one unit of work is a newton meter (abbreviated N · m). If you have measurements in newtons (a unit for force, F) and meters (a unit for distance, d), how would you calculate work? An analysis of the units will tell you that you should multiply force times distance to get work: $W = F \times d$.

Suppose you want to convert 0.0856 meters into millimeters. In this case, you need only one conversion factor that will cancel the meters unit and create the millimeters unit. The conversion factor will be created from the relationship 1000 mL = 1 m.

$$(0.0856 \text{ m}) \cdot \left(\frac{1000 \text{ mm}}{1 \text{ m}}\right) = (0.0856 \cancel{\text{m}}) \cdot \left(\frac{1000 \text{ mm}}{1 \cancel{\text{m}}}\right) = 85.6 \text{ mm}$$

In the above expression, the meter units will cancel and only the millimeter unit will remain.

Example:

Convert 1.53 g to cg.

The equivalency relationship is 1.00 g = 100 cg, so the conversion factor is constructed from this equivalency in order to cancel grams and produce centigrams.

$$(1.53 \text{ g}) \cdot \left(\frac{100 \text{ cg}}{1 \text{ g}}\right) = 153 \text{ cg}$$

Example:

Convert 1000. in. to ft.

The equivalency between inches and feet is 12 in. = 1 ft. The conversion factor is designed to cancel inches and produce feet.

$$(1000. \text{ in.}) \cdot \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 83.33 \text{ ft}$$

Each conversion factor is designed specifically for the problem. In the case of the conversion above, we need to cancel inches, so we know that the inches component in the conversion factor needs to be in the denominator.

Example:

Convert 425 *klums* to *piks* given the equivalency relationship 10 $klums = 1$ *pik*.

$$(425 \, klums) \cdot \left(\frac{1 \, pik}{10 \, klums}\right) = 42.5 \, piks$$

Sometimes, it is necessary to insert a series of conversion factors. Suppose we need to convert miles to kilometers, and the only equivalencies we know are 1 mi = 5,280 ft, 12 in. = 1 ft, 2.54 cm = 1 in., 100 cm = 1 m, and 1000 m = 1 km. We will set up a series of conversion factors so that each conversion factor produces the next unit in the sequence.

Example:

Convert 12 mi to km.

$$(12 \text{ mi}) \cdot \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) \cdot \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) \cdot \left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right) \cdot \left(\frac{1 \text{ m}}{100 \text{ cm}}\right) \cdot \left(\frac{1 \text{ km}}{1000 \text{ m}}\right) = 19 \text{ km}$$

In each step, the previous unit is canceled and the next unit in the sequence is produced.

Conversion factors for area and volume can also be produced by this method.

Example:

Convert 1500 cm^2 to m^2.

$$(1500 \text{ cm}^2) \cdot \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^2 = (1500 \text{ cm}^2) \cdot \left(\frac{1 \text{ m}^2}{10{,}000 \text{ cm}^2}\right) = 0.15 \text{ m}^2$$

Example:

Convert 12.0 in^3 to cm^3.

$$(12.0 \text{ in}^3) \cdot \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3 = (12.0 \text{ in}^3) \cdot \left(\frac{16.4 \text{ cm}^3}{1 \text{ in}^3}\right) = 197 \text{ cm}^3$$

Lesson Summary

- Conversion factors are used to convert one unit of measurement into another unit.
- Dimensional analysis involves considering both the units you presently have and the units you wish to end up with, as well as designing conversion factors that will cancel units you don't want and produce units you do want.

Further Reading / Supplemental Links

Visit this website for a video series that reviews topics on measurement

- http://www.learner.org/resources/series184.html

This website has lessons, worksheets, and quizzes on various high school chemistry topics. Lesson 2-4 is on dimensional analysis.

- http://www.fordhamprep.org/gcurran/sho/sho/lessons/lesson24.htm

Review Questions

1. For the equation $PV = nRT$, re-write it so that it is in the form of $T =$.
2. The equation for density is $D = \frac{m}{V}$. If D is 12.8 g/cm^3 and m is 46.1 g, solve for V, keeping significant figures in mind.
3. The equation $P_1 \cdot V_1 = P_2 \cdot V_2$, known as Boyle's law, shows that gas pressure is inversely proportional to its volume. Re-write Boyle's law so it is in the form of $V_1 =$.
4. The density of a certain solid is measured and found to be 12.68 g/mL. Convert this measurement into kg/L.
5. In a nuclear chemistry experiment, an alpha particle is found to have a velocity of $14,285$ m/s. Convert this measurement into miles/hour (mi/h).