Significant Figures

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CONCEPT 1 Significant Figures

Lesson Objectives

The student will:

- explain the necessity for significant figures.
- determine significant figures of the equipment pieces chosen.
- identify the number of significant figures in a measurement.
- use significant figures properly in measurements and calculations.
- determine the number of significant figures in the result of a calculation.
- round calculated values to the correct number of significant figures.

Vocabulary

significant figures includes all of the digits that can be known with certainty in a measurement plus an estimated last digit

Introduction

The numbers you use in math class are considered to be exact numbers. When you are given the number 2 in a math problem, it does not mean 1.999 rounded up to 2, nor does it mean 2.00001 rounded down to 2. In math class, the number 2 means exactly 2.000000... with an infinite number of zeros – a perfect 2! Such numbers are produced only by definition, *not* by measurement. We can define 1 foot to contain exactly 12 inches with both numbers being perfect numbers, but we cannot measure an object to be exactly 12 inches long. In the case of measurements, we can only read our measuring instruments to a limited number of subdivisions. We are limited by our ability to see smaller and smaller subdivisions, and we are limited by our ability to construct smaller and smaller subdivisions on our measuring devices. Even with the use of powerful microscopes to construct and read our measuring devices, we eventually reach a limit. Therefore, although the actual measurement of an object may be a perfect 12 inches, we cannot prove it to be so. Measurements do not produce perfect numbers; the only perfect numbers in science are defined numbers, such as conversion factors. Since measurements are fundamental to science, science does not produce perfect measurements.

It is very important to recognize and report the limitations of a measurement along with the magnitude and unit of the measurement. Many times, the measurements made in an experiment are analyzed for regularities. If the numbers reported show the limits of the measurements, the regularity, or lack thereof, becomes visible.

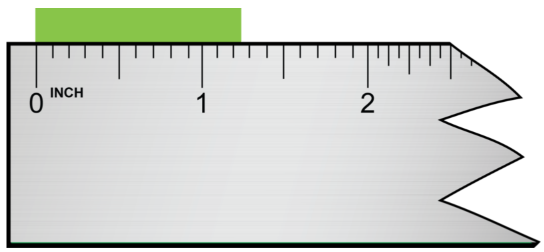
TABLE 1.1: Comparison of Observations with the Proper Number of Significant Figures

Observation List A	Observation List B
22.41359 m	22.4 m
22.37899 m	22.4 m
22.42333 m	22.4 m
22.39414 m	22.4 m

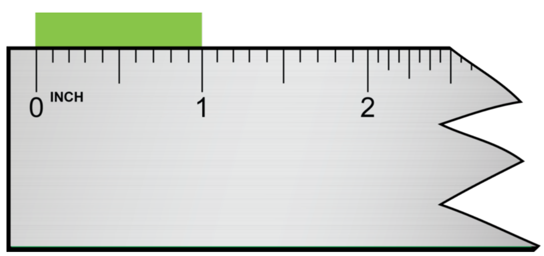
In the lists of observations shown in **Table** 1.1 , List A shows measurements without including the limits of the measuring device. In comparison, List B has the measurements rounded to reflect the limits of the measuring device. It is difficult to perceive regularity in List A, but the regularity stands out in List B.

Rules for Determining Significant Figures

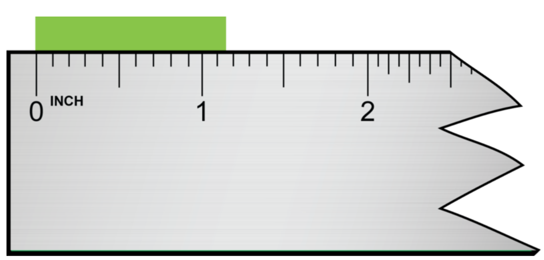
Significant figures, also known as significant digits, are all of the digits that can be known with certainty in a measurement plus an estimated last digit. Significant figures provide a system to keep track of the limits of the original measurement. To record a measurement, you must write down all the digits actually measured, including measurements of zero, and you must *not* write down any digit not measured. The only real difficulty with this system is that zeros are sometimes used as measured digits, while other times they are used to locate the decimal point.



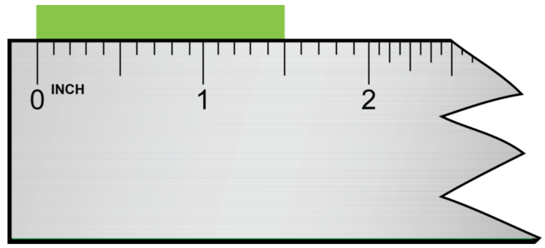
In the sketch shown above, the correct measurement is greater than 1.2 inches but less than 1.3 inches. It is proper to estimate one place beyond the calibrations of the measuring instrument. This ruler is calibrated to 0.1 inches, so we can estimate the hundredths place. This reading should be reported as 1.25 or 1.26 inches.



In this second case (sketch above), it is apparent that the object is, as nearly as we can read, 1 inch. Since we know the tenths place is zero and can estimate the hundredths place to be zero, the measurement should be reported as 1.00 inch. It is vital that you include the zeros in your reported measurement because these are measured places and are significant figures.



This measurement is read as 1.15 inches, 1.16 inches, or perhaps even 1.17 inches.



This measurement is read as 1.50 inches.

In all of these examples, the measurements indicate that the measuring instrument had subdivisions of a tenth of an inch and that the hundredths place is estimated. There is some uncertainty about the last, and only the last, digit.

In our system of writing measurements to show significant figures, we must distinguish between measured zeros and place-holding zeros. Here are the rules for determining the number of significant figures in a measurement.

Rules for Determining the Number of Significant Figures:

a. All non-zero digits are significant.
b. All zeros between non-zero digits are significant.
c. All beginning zeros are *not* significant.
d. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal (the decimal point is not written in).

Examples of the Significant Figure Rules:

1. All non-zero digits are significant.

543	has 3 significant figures.
22.437	has 5 significant figures.
1.321754	has 7 significant figures.

2. All zeros between non-zero digits are significant.

7,004	has 4 significant figures.
10.3002	has 6 significant figures.
103	has 3 significant figures.

3. All beginning zeros are *not* significant.

0.00000075	has 2 significant figures.
0.02	has 1 significant figure.
0.003003	has 4 significant figures.

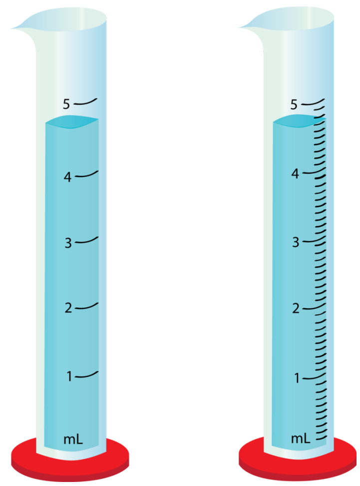
4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal.

37.300	has 5 significant figures.
33.00000	has 7 significant figures.
100.	has 3 significant figures.
100	has 1 significant figure.
302,000	has 3 significant figures.
1,050	has 3 significant figures.

Equipment Determines Significant Figures

Quality measuring instruments are made with as much consistency as possible and are individually calibrated after construction. In a graduated cylinder, for example, it is desirable for the sides to be perfectly vertical and for the inside diameter to be the same all the way up the tube. After the graduated cylinder is completed, exact volumes of liquids are placed in the cylinder and the calibration marks are then scribed onto the side of the tube.

The choice of measuring instrument determines the unit of measure and the number of significant figures in the measurement. Consider the two graduated cylinders shown below.



Both cylinders are marked to measure milliliters, but the cylinder on the left only shows graduations for whole milliliters. In comparison, the cylinder on the right has calibrations for tenths of milliliters. The measurer reads the volume from the calibrations and estimates one place beyond the calibrations. For the cylinder on the left, a reasonable reading is 4.5 mL. For the cylinder on the right, the measurer estimates one place beyond the graduations and obtains a reasonable reading of 4.65 mL. The choice of the measuring instrument determines both the units and the number of significant figures. If you were mixing up some hot chocolate at home, the cylinder on the left would be adequate. If you were measuring out a chemical solution for a very delicate reaction in the lab, however, you would need the cylinder on the right.

Similarly, the equipment chosen for measuring mass will also affect the number of significant figures. For example, if you use a pan balance (illustrated on the left in the image below) that can only measure to $\pm$ 0.1 g, you could only measure out 3.3 g of NaCl rather than 3.25 g. In comparison, the digital balance (illustrated on the right in the image below) might be able to measure to $\pm$ 0.01 g. With this instrument, you could measure what you need more exactly. The difference between these two balances has to do with the number of significant figures that the balances are able to measure. Whenever you need to make a measurement, make sure to check the number of significant figures a measuring instrument can measure before choosing an appropriate instrument.

Significant Figures in Calculations

In addition to using significant figures to report measurements, we also use them to report the results of computations made with measurements. The results of mathematical operations on measurements must indicate the number of significant figures in the original measurements. There are two rules for determining the number of significant figures after performing a mathematical operation. Most of the errors that occur in this area result from using the wrong rule, so always double check that you are using the correct rule for the mathematical operation involved.

Addition and Subtraction

The answer to an addition or subtraction operation must not have any digits further to the right than the shortest addend. In other words, the answer should have as many decimal places as the addend with the smallest number of decimal places.

Example:

```
  13.3843 cm
   1.012  cm
+  3.22   cm
  ----------
  17.6163 cm = 17.62 cm
```

Notice that the top addend has a 3 in the last column on the right, but neither of the other two addends have a number in that column. In elementary math classes, you were taught that these blank spaces can be filled in with zeros and the answer would be 17.6163 cm. In the sciences, however, these blank spaces are unknown numbers, *not* zeros. Since they are unknown numbers, you cannot substitute any numbers into the blank spaces. As a result, you cannot know the sum of adding (or subtracting) any column of numbers that contain an unknown number. When you add the columns of numbers in the example above, you can only be certain of the sums for the columns with known numbers in each space in the column. In science, the process is to add the numbers in the normal mathematical process and then round off all columns that contain an unknown number (a blank space). Therefore, the correct answer for the example above is 17.62 cm and has only four significant figures.

Example:

```
  12        m
+  0.00045  m
  -----------
  12.00045  m = 12 m
```

In this case, the addend 12 has no digits beyond the decimal. Therefore, all columns past the decimal point must be rounded off in the final answer. We get the seemingly odd result that the answer is still 12, even after adding a number to 12. This is a common occurrence in science and is absolutely correct.

Example:

```
   56.8885 cm
    8.30   cm
+  47.0    cm
  -----------
  112.1885 cm = 112.2 cm
```

Multiplication and Division

The answer for a multiplication or division operation must have the same number of significant figures as the factor with the least number of significant figures.

Example:

$$(3.556 \text{ cm}) \cdot (2.4 \text{ cm}) = 8.5344 \text{ cm}^2 = 8.5 \text{ cm}^2$$

The factor 3.556 cm has four significant figures, and the factor 2.4 cm has two significant figures. Therefore the answer must have two significant figures. The mathematical answer of 8.5344 cm^2 must be rounded back to 8.5 cm^2 in order for the answer to have two significant figures.

Example:

$$(20.0 \text{ cm}) \cdot (5.0000 \text{ cm}) = 100.00000 \text{ cm}^2 = 100. \text{ cm}^2$$

The factor 20.0 cm has three significant figures, and the factor 5.0000 cm has five significant figures. The answer must be rounded to three significant figures. Therefore, the decimal must be written in to show that the two ending zeros are significant. If the decimal is omitted (left as an understood decimal), the two zeros will not be significant and the answer will be wrong.

Example:

$$(5.444 \text{ cm}) \cdot (22 \text{ cm}) = 119.768 \text{ cm}^2 = 120 \text{ cm}^2$$

In this case, the answer must be rounded back to two significant figures. We cannot have a decimal after the zero in 120 cm^2 because that would indicate the zero is significant, whereas this answer must have exactly two significant figures.

Lesson Summary

- Significant figures are all of the digits that can be known with certainty in a measurement plus an estimated last digit.
- Significant figures provide a system to keep track of the limits of a measurement.
- Rules for determining the number of significant figures:

  1. All non-zero digits are significant.
  2. All zeros between non-zero digits are significant.
  3. All beginning zeros are *not* significant.
  4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal.

- The choice of measuring instrument is what determines the unit of measure and the number of significant figures in the measurement.
- The results of mathematical operations must include an indication of the number of significant figures in the original measurements.
- The answer for an addition or subtraction operation must not have any digits further to the right than the shortest addend.
- The answer for a multiplication or division operation must have the same number of significant figures as the factor with the least number of significant figures.

Further Reading / Supplemental Links

A problem set on unit conversions and significant figures.

- http://science.widener.edu/svb/pset/convert1.html

This website has lessons, worksheets, and quizzes on various high school chemistry topics. Lesson 2-3 is on significant figures.

- http://www.fordhamprep.org/gcurran/sho/sho/lessons/lesson23.htm

Review Questions

1. How many significant figures are in the following numbers?
    a. 2.3
    b. 17.95
    c. 9.89×10^3
    d. 170
    e. 22.1
    f. 1.02
    g. 19.84
2. Perform the following calculations and give your answer with the correct number of significant figures:
    a. $10.5 + 11.62$
    b. $0.01223 + 1.01$
    c. $19.85 - 0.0113$
3. Perform the following calculations and give your answer with the correct number of significant figures:
    a. 0.1886×12
    b. $2.995 \div 0.16685$
    c. $1210 \div 0.1223$
    d. 910×0.18945