Scientific Notation

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CONCEPT 1

Scientific Notation

Lesson Objectives

The student will:

- use scientific notation to express large and small numbers.
- add, subtract, multiply, and divide using scientific notation.

Vocabulary

scientific notation a shorthand method of writing very large and very small numbers by expressing them as a product of a decimal number between 1 and 10 multiplied by an integral power of 10

Introduction

Work in science frequently involves very large and very small numbers. The speed of light, for example, is 300,000,000 m/s; the mass of the earth is 6,000,000,000,000,000,000,000,000 kg; and the mass of an electron is 0.000000000000000000000000000009 kg. It is very inconvenient to write out such numbers and even more inconvenient to attempt to carry out mathematical operations with them. Scientists and mathematicians have designed an easier method to deal with such long numbers. This more convenient system is called exponential notation by mathematicians and scientific notation by scientists.

What is Scientific Notation?

In **scientific notation**, very large and very small numbers are expressed as the product of a number between 1 and 10 multiplied by some power of 10. For example, the number $9,000,000$ can be written as the product of 9 times $1,000,000$. In turn, $1,000,000$ can be written as 10^6. Therefore, $9,000,000$ can be written as 9×10^6. In a similar manner, 0.00000004 can be written as 4 times $\frac{1}{10^8}$, or 4×10^{-8}.

TABLE 1.1: Examples of Scientific Notation

Decimal Notation	Scientific Notation
95,672	9.5672×10^4

TABLE 1.1: (continued)

Decimal Notation	Scientific Notation
8,340	8.34×10^3
100	1×10^2
7.21	7.21×10^0
0.014	1.4×10^{-2}
0.0000000080	8.0×10^{-9}
0.00000000000975	9.75×10^{-12}

As you can see from the examples in **Table** 1.1, to convert a number from decimal form into scientific notation, you count the number of spaces needed to move the decimal, and that number becomes the exponent of 10. If you are moving the decimal to the left, the exponent is positive, and if you are moving the decimal to the right, the exponent is negative. You should note that *all significant figures are maintained in scientific notation*. You will probably realize that the greatest advantage of using scientific notation occurs when there are many non-significant figures.

Scientific Notation in Calculations

Addition and Subtraction

When numbers in exponential form are added or subtracted, the exponents must be the same. If the exponents are the same, the coefficients are added and the exponent remains the same.

Example:

$$(4.3 \times 10^4) + (1.5 \times 10^4) = (4.3 + 1.5) \times 10^4 = 5.8 \times 10^4$$

Note that the example above is the same as:

$$43,000 + 15,000 = 58,000 = 5.8 \times 10^4.$$

Example:

$$(8.6 \times 10^7) - (5.3 \times 10^7) = (8.6 - 5.3) \times 10^7 = 3.3 \times 10^7$$

Example:

$$(8.6 \times 10^5) + (3.0 \times 10^4) = ?$$

These two exponential numbers do not have the same exponent. If the exponents of the numbers to be added or subtracted are not the same, then one of the numbers must be changed so that the two numbers have the same exponent. In order to add them, we can change the number 3.0×10^4 to 0.30×10^5. This change is made by moving the decimal one place to the left and increasing the exponent by one. Now the two numbers can be added.

$$(8.6 \times 10^5) + (0.30 \times 10^5) = (8.6 + 0.30) \times 10^5 = 8.9 \times 10^5$$

We could also have chosen to alter the other number. Instead of changing the second number to a higher exponent, we could have changed the first number to a lower exponent.

8.6×10^5 becomes 86×10^4

$$(86 \times 10^4) + (3.0 \times 10^4) = (86 + 3.0) \times 10^4 = 89 \times 10^4$$

Even though it is not always necessary, the preferred practice is to express exponential numbers in proper form, which has only one digit to the left of the decimal. When 89×10^4 is converted to proper form, it becomes 8.9×10^5, which is precisely the same result as before.

Multiplication and Division

When multiplying or dividing numbers in exponential form, the numbers do not have to have the same exponents. To multiply exponential numbers, multiply the coefficients and add the exponents. To divide exponential numbers, divide the coefficients and subtract the exponents.

Multiplication Examples:

$$(4.2 \times 10^4) \cdot (2.2 \times 10^2) = (4.2 \cdot 2.2) \times 10^{4+2} = 9.2 \times 10^6$$

The product of 4.2 and 2.2 is 9.24, but since we are limited to two significant figures, the coefficient is rounded to 9.2.

$$(2 \times 10^9) \cdot (4 \times 10^{14}) = (2 \cdot 4) \times 10^{9+14} = 8 \times 10^{23}$$

$$(2 \times 10^{-9}) \cdot (4 \times 10^4) = (2 \cdot 4) \times 10^{-9+4} = 8 \times 10^{-5}$$

$$(2 \times 10^{-5}) \cdot (4 \times 10^{-4}) = (2 \cdot 4) \times 10^{(-5)+(-4)} = 8 \times 10^{-9}$$

$$(8.2 \times 10^{-9}) \cdot (8.2 \times 10^{-4}) = (8.2 \cdot 8.2) \times 10^{(-9)+(-4)} = 67.24 \times 10^{-13}$$

In this last example, the product has too many significant figures and is not in proper exponential form. We must round to two significant figures and adjust the decimal and exponent. The correct answer would be 6.7×10^{-12}.

Division Examples:

$$\frac{8 \times 10^7}{2 \times 10^4} = 4 \times 10^{7-4} = 4 \times 10^3$$

$$\frac{8 \times 10^{-7}}{2 \times 10^{-4}} = 4 \times 10^{(-7)-(-4)} = 4 \times 10^{-3}$$

$$\frac{4.6 \times 10^3}{2.3 \times 10^{-4}} = 2.0 \times 10^{(3)-(-4)} = 2.0 \times 10^7$$

In the example above, since the original coefficients have two significant figures, the answer must also have two significant figures. Therefore, the zero in the tenths place is written to indicate the answer has two significant figures.

Lesson Summary

- Very large and very small numbers in science are expressed in scientific notation.
- All significant figures are maintained in scientific notation.
- When numbers in exponential form are added or subtracted, the exponents must be the same. If the exponents are the same, the coefficients are added and the exponent remains the same.
- To multiply exponential numbers, multiply the coefficients and add the exponents.
- To divide exponential numbers, divide the coefficients and subtract the exponents.

Further Reading / Supplemental Links

This website has lessons, worksheets, and quizzes on various high school chemistry topics. Lesson 2-5 is on scientific notation.

- http://www.fordhamprep.org/gcurran/sho/sho/lessons/lesson25.htm

Review Questions

1. Write the following numbers in scientific notation.
   a. 0.0000479
   b. 251,000,000
   c. 4,260
   d. 0.00206

Do the following calculations without a calculator.

2. $(2.0 \times 10^3) \cdot (3.0 \times 10^4)$
3. $(5.0 \times 10^{-5}) \cdot (5.0 \times 10^8)$
4. $(6.0 \times 10^{-1}) \cdot (7.0 \times 10^{-4})$
5. $\frac{(3.0 \times 10^{-4}) \cdot (2.0 \times 10^{-4})}{2.0 \times 10^{-6}}$

Do the following calculations.

6. $(6.0 \times 10^7) \cdot (2.5 \times 10^4)$
7. $\frac{4.2 \times 10^{-4}}{3.0 \times 10^{-2}}$