# Normal Force and Friction Force

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# Concept 1 Normal Force and Friction Force

## Objectives

The student will:

- Understand how to solve problems involving the normal force.
- Understand how to solve problems involving friction.

## Vocabulary

- **friction:** A force that opposes two objects sliding against each other. Friction acts in a direction along the flat surface of an object.
- **normal force:** The force exerted by the ground or other object that prevents other objects from going through it.
- **vector components**

## Introduction

Just as velocity and acceleration are vectors that can point in any direction, force is a vector that has both a magnitude and a direction.

## The Normal Force

As discussed in one-dimensional forces, a **normal force** is the force exerted by the ground or other object that prevents other objects from going through it. In the **Figure** below, a block that rests upon the ground is pulled in a diagonal direction up and to the right. Just as velocity and acceleration can be $x$ and $y$ components, we will need the same skills in resolving the force vector in the **Figure** below.

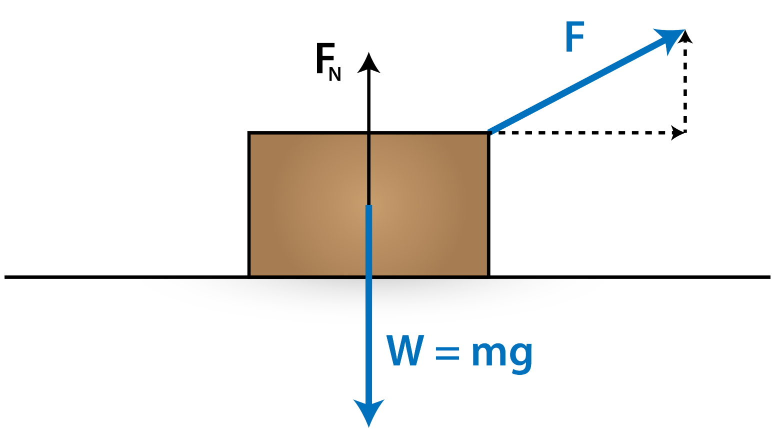


FIGURE 1.1

The applied force, $\vec{F}$, acts at an angle to the horizontal. Knowing the value of the angle is not necessary to understand the effect on the normal force. Recall that the normal force, $\vec{F}_N$, is the reaction force to the force that the block exerts on the ground. The applied force, $\vec{F}$, has two components. One component acts toward the right (the $x-$component) and the other component acts upward (the $y-$component). Consider the effect that the $y-$component has on the normal force. Since $y-$component of the force $\vec{F}$ acts in the upward direction, it effectively "eases" some of the block's weight off the ground. If the $y-$component were equal in magnitude to the weight of the block, the ground would not experience any force upon it due to the weight of the block. We can see that upward forces reduce the reaction force on the block. Thus, upward forces acting on the block reduce the normal force.

### Check Your Understanding

In the **Figure** above, the weight, $mg$, of the block is 100 N and the force, $\vec{F}$, has a $y-$component, $F_y$, of 25 N.

1. What normal force, $\vec{F}_N$, does the ground exert upon the block?

**Answer:** We know that gravity and the normal force act only in the $y-$direction, and that the block is not moving (velocity and acceleration zero). We can use Newton's Third Law, which can be applied to the net force in the $y-$direction and the acceleration in the $y-$direction.

$\sum_y F = F_N + F_y - mg = ma_y = 0.$

Therefore, $F_N = 100 - 25 = 75\ N$

### Ilustrative Examples

1a. A 25.0 kg block experiences an applied force $\vec{F}$, of 100 N acting at an angle 30° above the horizontal. What is the normal force, $F_N$, on the block?

**Answer:** The first thing to do is to resolve the applied force into its $x$ and $y$ components:

$$F_x = F\cos\theta = F\cos 30^\circ = 100\cos 30^\circ = 86.6\ N$$
$$F_y = F\sin\theta = F\sin 30^\circ = 100\sin 30^\circ = 50.0\ N$$

Next, since we were given the mass of the block, we need to find its weight:

$W = mg = (25.0\text{kg})(10\text{m/s}^2) = 250\text{ N}$

The question concerns the forces in the $y-$direction.

$\sum F_y = F_N + 50\text{N} - 250\text{N} = ma_y = 0$, therefore, $F_N = 200\text{ N}$

1b. What is the horizontal acceleration of the block in 1a?

**Answer:** This question concerns the forces in the $x-$direction.

$\sum F_x = 86.6\text{ N} = ma_x = (25.0\text{kg})a_x$, therefore, $a = 3.46\text{ m/s}^2$

### Check Your Understanding

Had the applied force been directed as shown in this diagram, what effect do you think it would have had on the normal force? See diagram below.

(a) The normal force is greater than the weight of the block.

(b) The normal force is less than the weight of the block.

(c) The normal force equals the weight of the block.

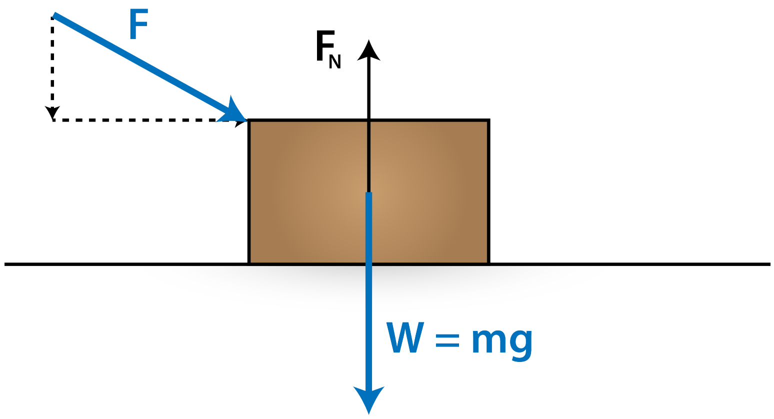


FIGURE 1.2

**Answer:** The answer is A. See if you can convince yourself that the normal force in this situation would be 295 N if the values of the given quantities were the same as in the previous question, but with the direction of $F$ as shown above.

## Friction

**Friction** is a force that opposes two objects sliding against each other, and is a contact force like the normal force. While the normal force acts perpendicular to the flat surface, friction acts in a direction along the flat surface of an object.

We generally speak of two kinds of friction: kinetic friction and static friction. We will begin our discussion of friction with kinetic friction.

## Kinetic Friction

"Kinetic" means *moving*. Kinetic friction means friction between two objects sliding against each other, such as: (1) sliding a book across a desktop and (2) your foot slipping on an icy pavement. The force from kinetic friction is abbreviated $f_k$. (Friction forces by convention use a lower case f.)

We know a force must exist on the book because it eventually stops moving. Newton's Second Law implies there must be some force acting on the book to slow it down and eventually bring it to rest. We call this force kinetic friction. Friction arises because no matter how smooth the surface of the book or the surface it is in contact with may look or feel, microscopically the two surfaces are rough. The smallest unevenness of the surfaces acts to impede the motion of the book. In fact, a force must be applied to the book just to overcome this "roughness" before it can be set into motion. The force that acts on the book before it is set into motion is called the static friction force, which we will discuss after dealing with kinetic friction.

Kinetic friction follows three basic rules:

1. The frictional force is independent of the relative velocity between the two surfaces for conventional speeds.
2. The frictional force is independent of contact surface area. If you slide a book lying flat or turn it on edge, the force of friction is the same.
3. The frictional force, $f_k$, is directly proportional to the normal force the two objects press against each other with, $F_N$, and also directly proportional to the roughness or stickiness of the surface, called its coefficient of friction. We write this as $f_k = \mu_k F_N$, where $\mu_k$ is the coefficient of friction. The symbol, $\mu$, is the Greek letter mu and pronounced "myoo" in English. The rougher or stickier the contact between two surfaces, the larger the value of $\mu_k$. A frictionless surface would have $\mu_k = 0$.

As you well know, it's easier to push an object from one point to another than to carry it from one point to another. We usually just accept this observation as obvious. But why is this so?

This observation leads to the conclusion that the force of kinetic friction is usually less than the weight of the object to be moved. If not, why push when you could more easily lift?

The coefficients of friction, $\mu_k$, are measured experimentally. A typical experimental set-up, which is often encountered in school physics laboratories, is to use a spring scale to pull increasing weights at a constant velocity, **Figure** below. Since the velocity is constant, the force that the spring scale exerts is equal to the magnitude of the kinetic friction. As the weight that is pulled increases, so too does the minimum force required to set the weight in motion. But as noted above, we would expect the force reading on the spring scale to be smaller than the weight being pulled. The ratio of the force on the spring scale to the weight of the object is found to be constant, and is $\mu_k$.

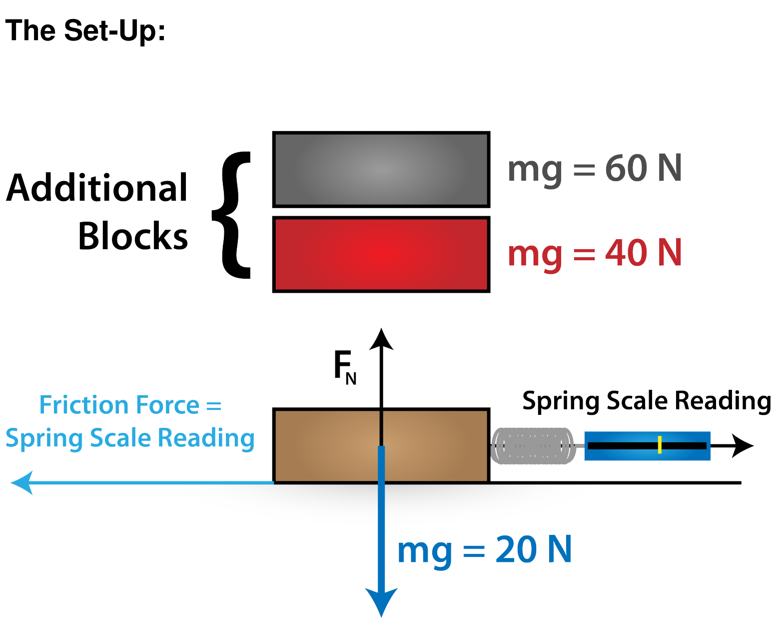


FIGURE 1.3

$mg = 20\ N$

TABLE 1.1:

| Weight (N) | Spring reading = friction force (N) |
|---|---|
| 20 | 10 |
| 40 | 20 |
| 60 | 30 |

If we graph kinetic friction force vs. weight, we would find a straight line with a slope of 0.5. The slope of this graph represents the coefficient of friction, $\mu_k$. Notice that friction is measured in Newtons since it is a force. What are the units of $\mu_k$?

**Figure** below shows two experiments measuring the coefficient of kinetic friction for brick and wood on polished oak.

### Check Your Understanding

1. What are the units of $\mu_k$?

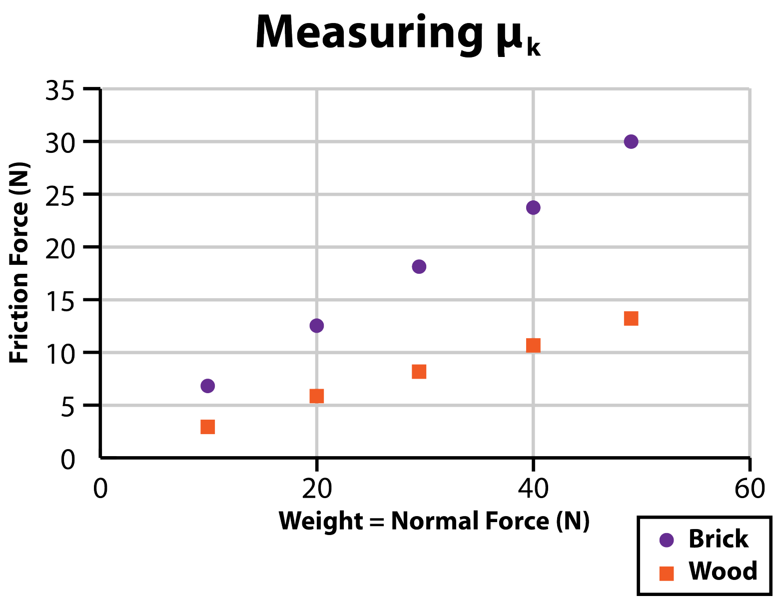


FIGURE 1.4

**Answer:** Since$\mu_k$is defined as the ratio$\frac{f_k}{F_N}$, it must be a "pure" number. It has no units since it is a ratio of force units:$\frac{N}{N}$which "cancel out."

2. It is typically stated that $\mu_k$ is less than 1 for common materials. There are exceptions, of course. Sliding rubber on rubber, for example, can result in $\mu_k > 1$. How can$\mu_k = 1$be interpreted if the motion takes place along a horizontal surface?

**Answer:** The force needed to slide the object is the same as the force needed to lift the object.

$f_k = (1)(F_N) = mg$

Since $u_k$ is usually found to be less than 1, it substantiates the notion that it's easier to push (or drag) an object than lift it!

### Illustrative Example 1

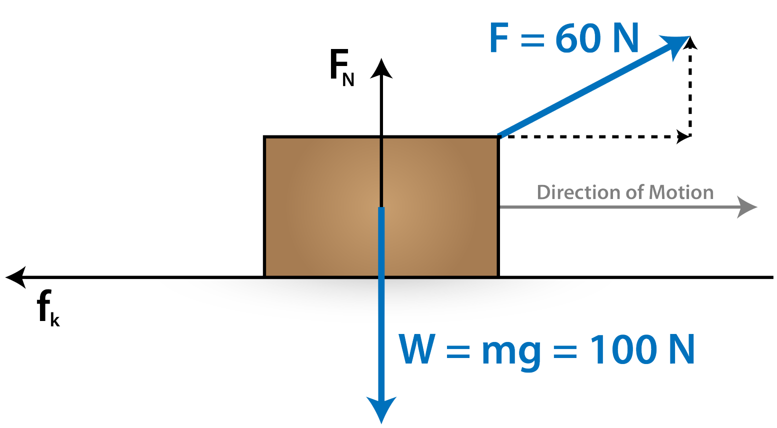


FIGURE 1.5

In **Figure** above, a force $F$ of 60 N acts at an angle of 30 degrees above the horizontal upon a block of weight 100

N moving it with constant velocity.

(a) Draw the Free Body Diagram (FBD) for the problem.

(b) Determine the friction force between the block and the surface.

(c) Determine the normal force on the block.

(d) Determine the coefficient of kinetic friction, $\mu_k$.

**Answers:**

(a) See **Figure** above. Notice how the friction force is represented in FBDs. It is drawn as an "interface" vector.

(b) Since the block moves at constant velocity $\sum_x F = 0$.

Therefore $\sum_x F = F\cos\theta - f_k = 0, 60\cos 30^\circ = f_k; f_k = 51.96 = 52\ N$.

(c) We assume the block has no vertical motion, therefore, $\sum_y F = 0$

$\sum_y F = F_N + F\sin\theta - mg = 0; F_N = mg - F\sin\theta = 100 - 60\sin 30^\circ = 70\ N$

(d) Since $f_k = \mu_k F_N; \mu_k = \frac{f_k}{F_N} = \frac{52}{70} = 0.74; \mu_k = 0.74$

### Static Friction

The original meaning of "static" is "not moving". Static friction, $f_s$, exists when the contact surfaces do not slide relative to one another. Two examples would be: (1) a coin on an inclined surface that remains stationary and (2) exerting a force on a heavy couch that refuses to move.

For a given pair of surfaces, the coefficient of static friction $\mu_s$ is larger than the coefficient of kinetic friction, $\mu_k$. Put simply, there is less friction when objects are in motion. We mentioned earlier in the section that regardless of how smooth the surfaces of two objects appear, at the microscopic level they are very rough. Atoms actually interact along the irregular contact areas between the surfaces forming electrical bonds. As soon as there is relative motion between the surfaces, the bonds begin to break. Heat due to friction continues to aid in breaking the bonds, making it easier to maintain motion. Kinetic friction is smaller than static friction; and $\mu_k < \mu_s$.

Consider **Figure** below.

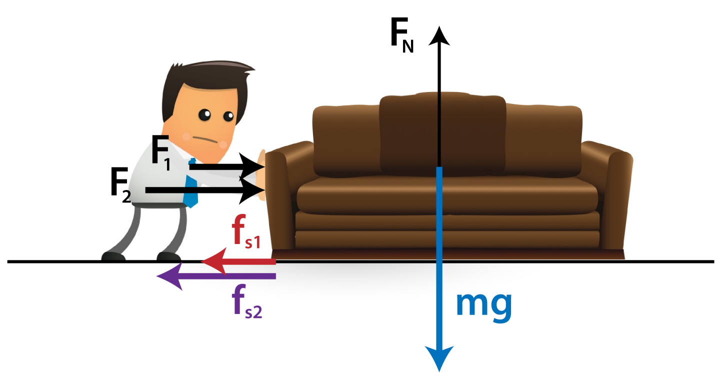


FIGURE 1.6

The man in the figure is trying to slide a heavy couch. He exerts a force $F_1$ which is insufficient to set the couch in motion. He then applies a greater force $F_2$ and the couch still does not move. In each case, since a force was placed upon the couch, and it remained stationary, an equal and opposite force must have acted upon the couch ($f_{s1}$ and $f_{s2}$, respectively) such that the net force on the couch remained zero, and the couch remained at rest. We call this force static friction. But unlike kinetic friction, the static friction force is not confined to one value. For example, if $F_1$ was 100 N and $F_2$ was 150 N in **Figure** above, then the static friction forces were $f_{s1} = 100\ N$ and $f_{s2} = 150\ N$, respectively. In fact, the static friction force can take on any value greater than or equal to zero up to the maximum force at which the couch is set into motion. At the point the couch is set into motion, static friction is gone and kinetic friction begins. Because static friction can take on any value up to the point of motion, we define static friction using an inequality: $f_s \leq \mu_s F_N$

The coefficient of static friction, $\mu_s$, is found by determining the maximum force, $f_{s\ max}$, just before the instant an object is set into motion. We will generally drop the subscript on the static friction force when the context is clear.

### Illustrative Example 2

The couch in **Figure** above just begins to move when a force of 175 N is applied to it.

(a) What is the maximum static friction force, $f_s$, between the couch and the floor?

(b) What is the coefficient of static friction, $\mu_s$, if the couch weighs 1000 N?

**Answers:**

(a) Since the maximum force applied before the couch moves is 175 N, this must be the maximum static friction: $f_s = 175\ N$.

(b) $\mu_s = \frac{f_s}{F_N} = \frac{175N}{1000N} = 0.175$

Notice that the coefficient of static friction, $\mu_s$, is a pure number (it has no units) just as the coefficient of kinetic friction, $\mu_k$. This is because the coefficient of static friction, $\mu_s$, depends upon the nature of the materials in contact and it is a ratio of two forces, as is the coefficient of kinetic friction, $\mu_k$.

Kinetic and static friction oppose motion. Friction acts to oppose the motion caused by an applied force, thus opposing the relative motion between two surfaces.

If you attempt to accelerate your car and there is insufficient static friction between the tires and the road (for example, if you're on ice), the tires would spin and the car would gain no additional speed. Kinetic friction would oppose the motion of the tires, even on ice, and you'd "burn rubber." However, during those moments when your tires made contact with the asphalt, static friction would oppose the applied force your tires put upon the road and send the car forward. At the area of contact between the tire and the road, the tire pushes back on the pavement and the pavement pushes on the tire in the forward direction (Newton's Third Law in action!). The force of static friction is responsible for pushing the car forward. The force of static friction opposes the motion of the tire relative to the road, but has the same direction as the velocity of the car.

### Check Your Understanding

1. How does the magnitude of the force of friction change as the angle of $F$ increases from the horizontal?

**Answer**: As the angle increases, the normal force decreases so the friction force must also decrease. If the normal force goes to zero, so does the friction force.

2. If the man in the picture were taller and he applied the force of the same magnitude to the weight, how would the normal force on the weight change?

**Answer**: The taller he is, the more vertical the force, so the normal force would decrease.

3. A student pushes a calculator along a table with a horizontal force of 1 N, but the calculator remains motionless.

FIGURE 1.7

(a) What is the magnitude of the static friction force on the calculator?

(b) Can the static friction force ever be smaller or larger than 1 N?

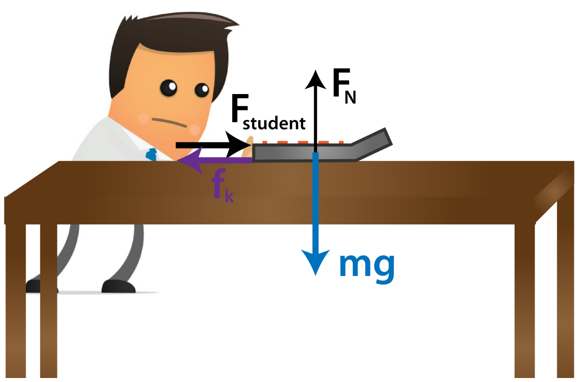


FIGURE 1.8

http://phet.colorado.edu/en/simulation/forces-and-motion

**Answers**:

2a. Since the calculator remains stationary, the force of static friction and the force that the student exerts must be equal and opposite.

2b. The static force can easily be smaller. Any force less than 1 N that the student exerts upon the calculator will be equal and opposite to the static friction force. The static friction can take on an infinite number of values below the threshold force that sets the calculator in motion.

The static friction force may possibly be larger than 1 N if the student applies a bit more force and the calculator remains motionless. The force at which the calculator just begins to move is the maximum force that static friction can provide.

## References


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