Newton's Third Law

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CONCEPT 1 Newton's Third Law

- Understand Newton's Third Law
- Understand the difference between countering force and action-reaction
- Use Newton's Three Laws to solve problems in one dimension

Vocabulary

- **center of mass**: The point at which all of the mass of an object is concentrated.
- **dynamics**: Considers the forces acting upon objects.
- **Free-Body-Diagram (FBD)**: A diagram that shows those forces that act upon an object/body.

Equations

$\sum F = Ma$

Newton's Third Law: Forces or Pairs of Forces

It was Newton who realized singular forces could not exist: they must come in pairs. In order for there to be an "interaction" there must be at least two objects, each "feeling" the other's effect.

Newton's Third Law: Whenever two objects interact, they must necessarily place equal and opposite forces upon each other.

Mathematically, Newton's Third law is expressed as $F_{AB} = -F_{BA}$, where the subscript AB means the force exerted on A by B and the subscript BA means the force exerted on B by A. Forces F_{AB} and F_{BA} are identical forces and never act upon the same object. Forces that are equal and opposite and act upon the same object are not a pair.

Problem Solving

We use Newton's laws to solve **dynamics** problems. Dynamics, unlike kinematics, considers the forces acting upon objects. Whether it is a system of stars gravitationally bound together or two colliding automobiles, we can use Newton's laws to analyze and quantify their motion. Of Newton's three laws, the major mathematical "workhorse" used to investigate these and endless other physical situations is Newton's Second Law (N2L): $\sum F = Ma$.

In using Newton's laws, we assume that the acceleration is constant in all of the examples in the present chapter. Newton's laws can certainly deal with situations where the acceleration is not constant, but for the most part, such situations are beyond the level of this book. A notable exception to this is when we investigate oscillatory motion. As a last simplification we assume that all forces act upon the **center of mass** of an object. The center of mass of an object can be thought of as that point where all of the mass of an object is concentrated. if your finger were placed at this point, the object would remain balanced. The 50 cm point is, for example, the center of mass of a meter stick.

Free-Body-Diagrams

A diagram showing those forces that act upon a body is called a **Free-Body-Diagram (FBD)**. The forces in a FBD show the direction in which each force acts, and, when possible, the relative magnitude of the each force by the length of the force vector. Each force in a FBD must be labeled appropriately so it is clear what each arrow represents.

Example 1: Sitting Bull

In the **Figure** below, a 1.0 kg bull statue is resting on a mantelpiece. Analyze the forces acting on the bull and their relationship to each other. There are two vertical forces that act upon the bull:

1. The Earth pulling down on the center of mass of the bull with a force of $W = mg = (1.0)(9.8) = 9.8\ N$
2. The floor pushing back against the weight of the bull, with a normal force F_N. The term normal force comes from mathematics, where normal means that the force is perpendicular to a surface. The normal force vector (often stated as "the normal") is drawn perpendicular to the surface that the bull rests upon. Normal forces are usually associated with a push upon an object, not a pull.

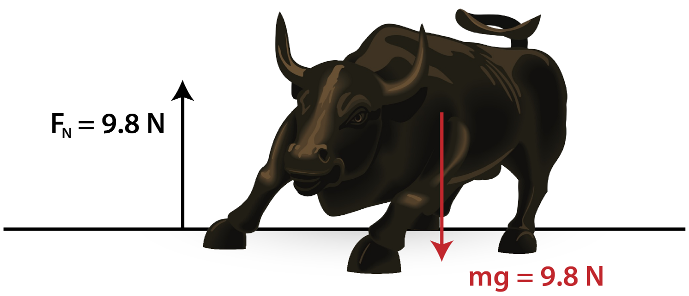

FIGURE 1.1

Answer: Using N2L we write: $\sum F = Ma$, where $\sum F = F_N - mg = ma = 0$, where $a = 0$. The negative sign $(-mg)$ indicates that the Earth pulls on the statue downward. Usually, when solving problems with N2L, forces that point down and to the left are expressed negatively and forces that point up and to the right are expressed positively. These are just conventions and any consistent set of conventions is permissible. It is also important (when enough information is provided) to draw the length of a vector in proportion to its magnitude. In the diagram above, F_N and mg are drawn the same length, reflecting the fact that they have the same magnitude. Important: in the diagrams, the arrows must originate inside the object, pointing "outward"

The statue is stationary so it has zero acceleration. This reduces the problem to $F_n = mg$, which intuitively seems reasonable. When the problem is solved, it shows the magnitudes of the forces are equal. It must be kept in mind that their directions are opposite and that they are not a N3L pair.

Example 2: Hanging Loose

In the diagram below Mr. Joe Loose is hanging from a rope for dear life. Joe's mass is 75 kg. Use $g = 9.8\ m/s^2$.

2a. Draw Joe's FBD

2b. What is the tension in the rope?

Answer: We assume the mass of the rope is negligible. Including the mass of the rope is not particularly difficult, but we're just starting out!

The convention in physics is to use label T, for "tension". A tension force is transmitted through a string, cord, or rope.

FIGURE 1.2

Once again, we apply $\sum F = Ma$, where $\sum F = T - mg = ma = 0$ since $a = 0$.

$T = mg = (75.0)(9.8) = 735\ N = 740\ N$

Example 3: Sliding Away

A 4900 N block of ice, initially at rest on a frictionless horizontal surface, has a horizontal force of 100 N applied to it.

Answer: Always begin by drawing an FBD of the problem.

Typically, applied forces are either written as F or F_{ab}. If there are multiple forces, depending on the wording of the problem, each force may have a subscript that reflects its meaning, or may just be numbered.

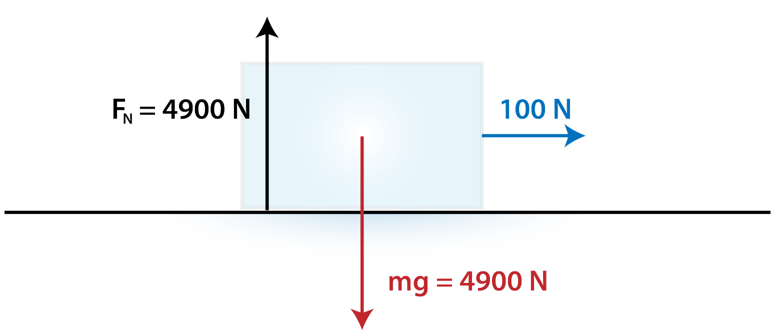

FIGURE 1.3

3a. Find the mass of the block of ice in **Figure** above, use 9.8 m/s^2 for g.

Answer: $W = mg,\ m = \frac{4900\ N}{9.8\ m/s^2} = 500\ kg$

3b. Find the acceleration of the block of ice in **Figure** above.

Answer: $\sum F = Ma,\ \sum F = F_{ap} = ma.$

$100\ N = (500\ kg)a,\ a = 0.20\ m/s^2$

3c. Find the velocity of the block at $t = 100\ s$

Answer: $V_f = at + V_i,\ (0.20)(100) + 0 = 20\ m/s$

3d. Find the displacement of the block at $t = 100\ s$.

Answer: $\Delta x = \frac{1}{2}(V_i + V_f)t = \frac{1}{2}(0 + 20)(100) = 1000\ m$

Example 4: A Touching Story

In **Figure** below, Block A has a mass of 10.00 kg and Block B has a mass of 6.00 kg. Both blocks are in contact with each other, with Block A experiencing an applied 70.0 N force to the right as shown. Note that both blocks have the same acceleration.

Note: When referring to more than one mass we often use the word "system."

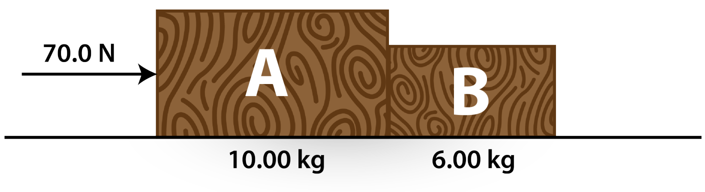

FIGURE 1.4

4a. Draw the FBD's for Block A and Block B

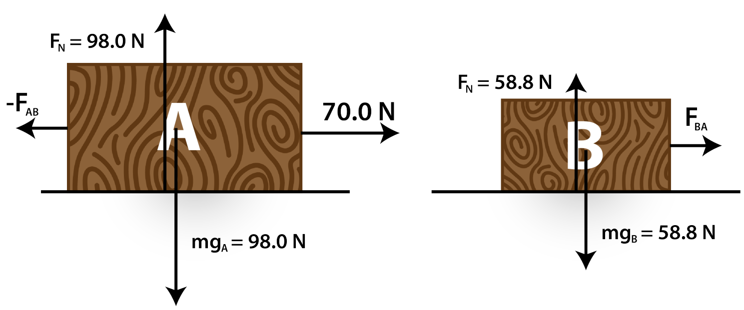

FIGURE 1.5

Answer: As Block$A$moves to the right it experiences a force from Block$B$to the left.

This force is labeled: $-F_{AB}$ (force on A by B)

Block B is pushed to the right with the same force that it exerts upon block A, according to N3L

This force is labeled: F_{BA} (force B by A); the magnitudes of F_{AB} and F_{BA} are, of course, equal, according to N3L.

4b. Find the acceleration of the system.

Answer: We use N2L applied to each block:

Block A : $\sum f = MA$. $\sum F = 70.0 - F_{AB} = 10.00a$.

Block B : $\sum F = MA$. $\sum F = F_{BA} = 6.00a$.

We have a system of two equations and two unknowns since the magnitudes of F_{AB} and F_{BA} are identical.

Adding F_{BA} to left side of the Block A equation and $6.00a$ to the right side of it, we have:

$70.0 = 16.00a,\ a = 4.375 = 4.38\ m/s^2$.

Notice that on the left side of the resulting equation, the sum of $-F_{AB}$ and F_{BA} is zero and on the right side of the equation the sum is $10.00a + 6.00a$. The N3L pair of forces, $-F_{AB}$ and F_{BA} is considered as a pair of internal forces with respect to the system. When solving a system of equations having an N3L pair of forces, these internal forces add up to zero. Additionally, the right-hand side of the equation must always equal the total mass of the system. For this example $(m_A + m_B)a$. In more complicated problems, care must be taken if different parts of the system have different accelerations.

4c. What is the magnitude of the force between Block A and Block B (F_{AB} or F_{BA})?

Answer: This is answered by solving either the Block$A$or Block$B$equation. The Block$B$equation is certainly easy to solve. Dropping the subscript:$F = 6.00(4.375) = 26.25 = 26.3\ N$

The Atwood Machine

The Atwood Machine (invented by English mathematician Reverend George Atwood, 1746-1807) is used to demonstrate Newton's Second Law, notably in determining the gravitational acceleration, g.

Example 5:

One end of the rope in **Figure** below is attached to a 3.2-kg mass, m_1 and the other end is attached to a 2.0-kg mass m_2. Assume the system is frictionless and the rope has negligible mass.

5a. Draw FBDs for the m_1 and m_2.

Answer:

5b. Determine the acceleration of the system. Use $g = 9.8\ m/s^2$.

Answer: Before we begin, we decide in which direction the system accelerates. Since the mass of the$m_2$is smaller than the mass ofm_1, m_2will accelerate up and$m_1$will accelerate down. Therefore the tension in the rope is greater than the weight of$m_1$but smaller than the weight ofm_2. Using N2L we write the equations of motion for$m_1$andm_2.

$\sum F = (3.2)(9.8) - T = 32a$ (since $T < mg_1, a > 0$)

$\sum F = T - (2.0)(9.8) - T = 2.0a$ (since $T < mg_2, a > 0$)

The equations are set up so that the acceleration has a consistent sign.

Had we chosen the direction of the acceleration incorrectly, our answer would have been a negative number, informing us of our error.

Solving the system of equations we have:

$(3.2)(9.8) - (2.0)(9.8) = 5.2a$, solving for the acceleration gives: $a = 2.26\ m/s^2 = 2.3\ m/s^2$.

5c. Find the tension in the rope.

Answer: Again, either equation will provide the answer. Using the second equation above, we have:

$T = (2.0)(9.8) + (2.0)(2.26) = 24.12\ N = 24\ N$

One possible check on the problem is to insure that: $mg_2 < T < mg_1$

$mg_{ax} = (2.0)(9.8) = 19.6\ N = 20\ N$ and $mg_{log} = (3.2)(9.8) = 31.36\ N = 31\ N$.

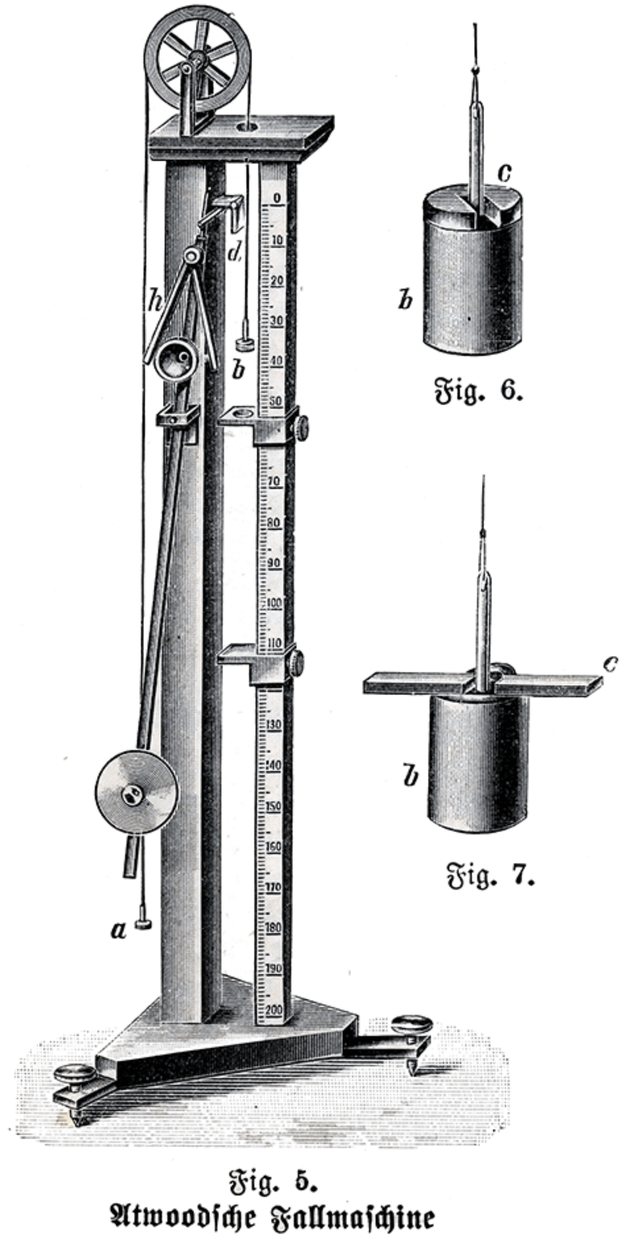

FIGURE 1.6

Atwood used a pendulum mechanism for timing the system of masses and adjusted the distances of the masses to ensure an integral number of seconds.

Therefore: $20 < 24 < 31$.

It is always wise to check your results for consistency.

http://www.youtube.com/watch?v=NnYG2cnRwgQ

References

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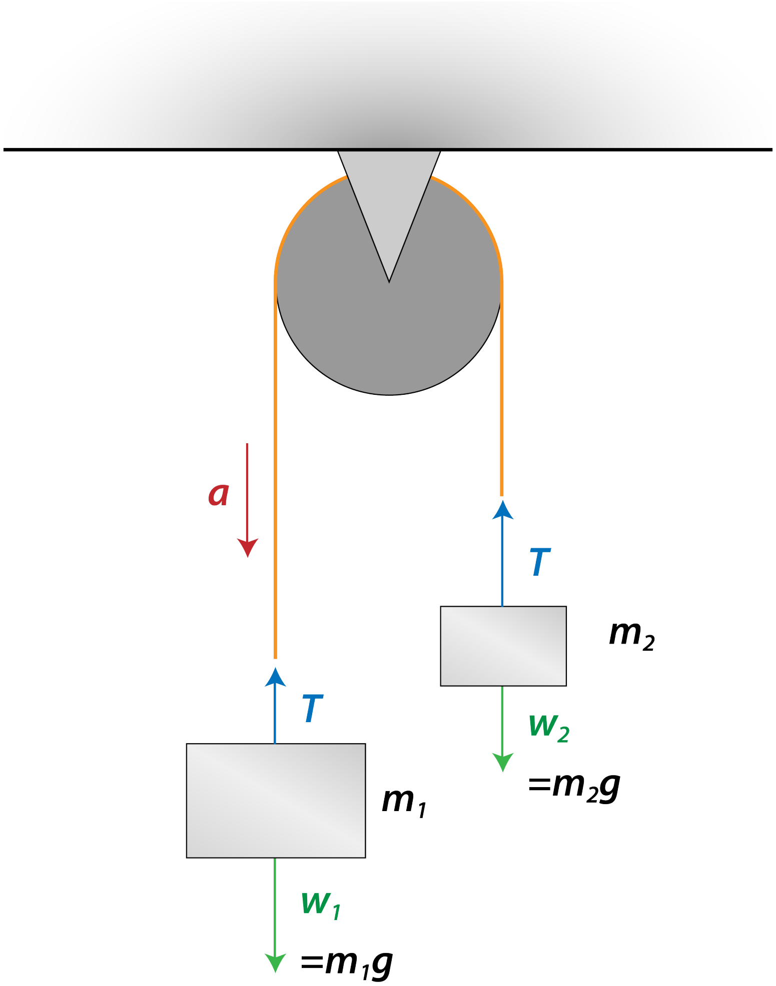

FIGURE 1.7