

# Mathematics Tools for Physics

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## CONCEPT

## 1

# Mathematics Tools for Physics

- Convert given numbers from expanded form to exponential form and vice versa.
- Carry out mathematical operations with exponential numbers.
- Identify the number of significant figures in given measurements.
- Carry proper significant figures through math operations.



Scientists use computers to aid with all the mathematics needed to do science. Computers take and store measurement readings and do hours of mathematical calculations. The image is of the CSIRAC (Council for Scientific and Industrial Research Automatic Computer), Australia's first digital computer, first run in November 1949. This room-filling computer was able to do less than the personal computers of today.

## Mathematics Tool Kit

### What is the place of mathematics in physics?

It has been said by beginning physics students that physics is just another math class. Well, there is a lot of math in physics but none of the concepts and theories of physics can be derived strictly from mathematics. If you wish to build a large complex mechanical structure, it is pointless to begin without wrenches and screwdrivers. To build mechanical structures, you must have tools. One of the primary tools for working in physics is mathematics. At the high school level, physics requires algebra, geometry, and trigonometry. If you go to higher-level physics, calculus is also required. Calculus was, in fact, invented by Isaac Newton specifically for doing physics. If you study higher mathematics, you will find, when you look at the practice problems at the end of the chapter of the math book, you are looking at physics problems.

## Exponential Notation

In the definition of physics, it was noted that physics deals with objects as small as sub-atomic particles and as large as galaxies. It should be clear that physicists deal with extremely small numbers like the mass of a lead atom, 0.000000000000000000000000000034 gram, and extremely large numbers like the distance from our galaxy to the Andromeda galaxy, 2.5 million light years, which is 25,000,000,000,000,000 kilometers.

These numbers are difficult to write and even more difficult when calculations must be done. It is much more convenient to write and calculate with such extreme numbers if they are written in exponential form. In exponential form, the mass of a lead atom is  $3.4 \times 10^{-34}$  g, and the distance from our galaxy to the Andromeda galaxy is  $2.5 \times 10^{19}$  km.

A number is expressed in exponential form by moving the decimal so that exactly one non-zero digit is on the left of the decimal and the exponent of 10 will be the number of places the decimal was moved. If the decimal is moved to the left, the exponent is positive and if the decimal was moved to the right, the exponent is negative. All **significant figures** are maintained in **exponential notation**.

**Example:** Express 13,700,000,000 in exponential form.

**Solution:** Since the decimal will be moved to the left 10 places, the exponent will be 10. So, the correct exponential form is  $1.37 \times 10^{10}$ .

**Example:** Express 0.00000000000000000000000074 in exponential form.

**Solution:** Since the decimal will be moved to the right 17 places, the exponent will be -17. So the correct exponential form is  $7.4 \times 10^{-17}$ .

**Example:** Express the number  $8.43 \times 10^5$  in expanded form.

**Solution:**  $10^5$  is 100,000 so  $8.43 \times 10^5$  is  $8.43 \times 100,000$  or 843,000.

## Operations with Exponential Numbers

In order to add or subtract exponential numbers, the exponents must be the same. If the exponents are not the same, one of the numbers must be changed so that the exponents are the same. Once the exponents are the same, the numbers are added and the same exponents are carried.

**Example:** Add  $5.0 \times 10^5$  and  $4.0 \times 10^4$ .

**Solution:** In order to add these numbers, we can change  $4.0 \times 10^4$  to  $0.40 \times 10^5$  and then add  $0.40 \times 10^5$  to  $5.0 \times 10^5$  which yields  $5.4 \times 10^5$ .

When you multiply exponential numbers, the numbers multiply and the exponents add.

**Example:** Multiply  $5.0 \times 10^5$  and  $4.0 \times 10^4$ .

**Solution:**  $(5.0 \times 10^5)(4.0 \times 10^4) = (5.0)(4.0) \times 10^{5+4} = 20 \times 10^9 = 2 \times 10^{10}$

**Example:** Multiply  $6.0 \times 10^3$  and  $2.0 \times 10^{-5}$ .

**Solution:**  $(6.0 \times 10^3)(2.0 \times 10^{-5}) = 12 \times 10^{3-5} = 12 \times 10^{-2} = 1.2 \times 10^{-1} = 0.12$

When you divide exponential numbers, the numbers are divided and the exponent of the divisor is subtracted from the exponent of the dividend.

**Example:** Divide  $6.0 \times 10^3$  by  $2.0 \times 10^{-5}$ .

**Solution:**  $\frac{6.0 \times 10^3}{2.0 \times 10^{-5}} = 3.0 \times 10^{3-(-5)} = 3.0 \times 10^8$

## Significant Figures

The numbers you use in math class are considered to be exact numbers. When you are given the number 2 in a math problem, it does not mean 1.999 rounded up to 2, nor does it mean 2.00001 rounded down to 2. In math class, the number 2 means exactly 2.000000... with an infinite number of zeros – a perfect 2! Such numbers are produced only by definition, *not* by measurement. We can define 1 foot to contain exactly 12 inches with both numbers being perfect numbers, but we cannot measure an object to be exactly 12 inches long. In the case of measurements, we can only read our measuring instruments to a limited number of subdivisions. We are limited by our ability to see smaller and smaller subdivisions, and we are limited by our ability to construct smaller and smaller subdivisions on our measuring devices. Even with the use of powerful microscopes to construct and read our measuring devices, we eventually reach a limit. Therefore, although the actual measurement of an object may be a perfect 12 inches, we cannot prove it to be so. Measurements do not produce perfect numbers; the only perfect numbers in science are defined numbers, such as conversion factors.

It is very important to recognize and report the limitations of a measurement along with the magnitude and unit of the measurement. Many times, the measurements made in an experiment are analyzed for regularities. If the numbers reported show the limits of the measurements, the regularity, or lack thereof, becomes visible.

Consider the following table of the pressures and volumes of a gas sample and the calculated PV product.

**TABLE 1.1:**

<u>Pressure</u>	<u>Volume</u>	<u>Pressure <math>\times</math> Volume (P <math>\times</math> V)</u>
4.01 atm	6.03 L	24.1803 L-atm
3.02 atm	7.99 L	24.1298 L-atm
6.04 atm	3.98 L	24.0392 L-atm
11.98 atm	1.99 L	23.8402 L-atm

Now look at this same set of data when we are told that all the measurements have only two **significant figures** and all the numbers must be rounded to two places.

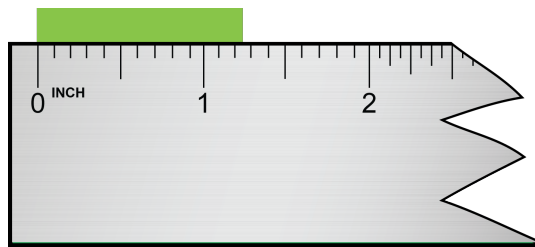
**TABLE 1.2:**

<u>Pressure</u>	<u>Volume</u>	<u>Pressure <math>\times</math> Volume (P <math>\times</math> V)</u>
4.0 atm	6.0 L	24 L-atm
3.0 atm	8.0 L	24 L-atm
6.0 atm	4.0 L	24 L-atm
12 atm	2.0 L	24 L-atm

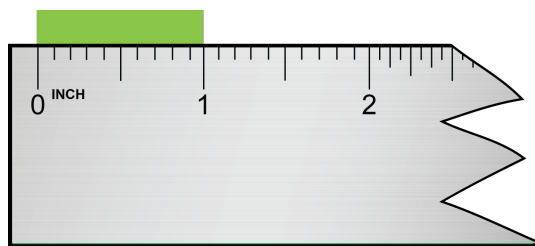
When the numbers are expressed with proper number of significant figures, a regularity appears that was not apparent before.

## Rules for Determining Significant Figures

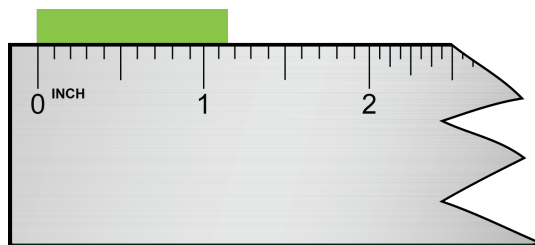
Significant figures are all of the digits that can be known with certainty in a measurement plus an estimated last digit. Significant figures provide a system to keep track of the limits of the original measurement. To record a measurement, you must write down all the digits actually measured, including measurements of zero, and you must *not* write down any digit not measured. The only real difficulty with this system is that zeros are sometimes used as measured digits, while other times they are used to locate the decimal point.



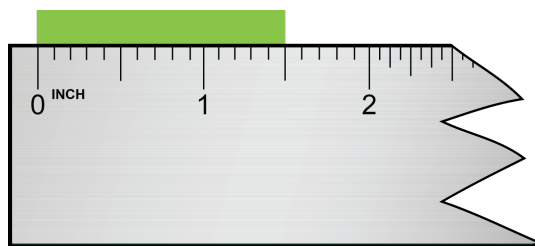
In the sketch shown above, the correct measurement is greater than 1.2 inches but less than 1.3 inches. It is proper to estimate one place beyond the calibrations of the measuring instrument. This ruler is calibrated to 0.1 inches, so we can estimate the hundredths place. This reading should be reported as 1.25 or 1.26 inches.



In this second case (sketch above), it is apparent that the object is, as nearly as we can read, 1 inch. Since we know the tenths place is zero and can estimate the hundredths place to be zero, the measurement should be reported as 1.00 inch. It is vital that you include the zeros in your reported measurement because these are measured places and are significant figures.



This measurement is read as 1.15 inches, 1.16 inches, or perhaps even 1.17 inches.



This measurement is read as 1.50 inches.

In all of these examples, the measurements indicate that the measuring instrument had subdivisions of a tenth of an inch and that the hundredths place is estimated. There is some uncertainty about the last, and only the last, digit.

In our system of writing measurements to show significant figures, we must distinguish between measured zeros and place-holding zeros. Here are the rules for determining the number of significant figures in a measurement.

**Rules for Determining the Number of Significant Figures:**

1. All non-zero digits are significant.
2. All zeros between non-zero digits are significant.
3. All beginning zeros are *not* significant.
4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal (the decimal point is not written in).

**Examples of the Significant Figure Rules:**

1. All non-zero digits are significant.  
543 has 3 significant figures.  
22.437 has 5 significant figures.  
1.321754 has 7 significant figures.
2. All zeros between non-zero digits are significant.  
7, 004 has 4 significant figures.  
10.3002 has 6 significant figures.  
103 has 3 significant figures.
3. All beginning zeros are *not* significant.  
0.0000075 has 2 significant figures.  
0.02 has 1 significant figure.  
0.003003 has 4 significant figures.
4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal.  
37.300 has 5 significant figures.  
33.00000 has 7 significant figures.  
100. has 3 significant figures.  
100 has 1 significant figure.  
302, 000 has 3 significant figures.  
1, 050 has 3 significant figures.

**Significant Figures in Addition and Subtraction**

The answer to an addition or subtraction operation must not have any digits further to the right than the shortest addend. In other words, the answer should have as many decimal places as the addend with the smallest number of decimal places.

**Example:**

$$\begin{array}{r}
 13.3843 \text{ cm} \\
 1.012 \text{ cm} \\
 + 3.22 \text{ cm} \\
 \hline
 17.6163 \text{ cm} = 17.62 \text{ cm}
 \end{array}$$

Notice that the top addend has a 3 in the last column on the right, but neither of the other two addends have a number in that column. In elementary math classes, you were taught that these blank spaces can be filled in with zeros and the answer would be 17.6163 cm. In the sciences, however, these blank spaces are unknown numbers, *not* zeros. Since they are unknown numbers, you cannot substitute any numbers into the blank spaces. As a result, you cannot know the sum of adding (or subtracting) any column of numbers that contain an unknown number. When you add the columns of numbers in the example above, you can only be certain of the sums for the columns with known numbers in each space in the column. In science, the process is to add the numbers in the normal mathematical process and then round off all columns that contain an unknown number (a blank space). Therefore, the correct answer for the example above is 17.62 cm and has only four significant figures.

## Multiplication and Division

The answer for a multiplication or division operation must have the same number of significant figures as the factor with the least number of significant figures.

**Example:**  $(3.556 \text{ cm})(2.4 \text{ cm}) = 8.5344 \text{ cm}^2 = 8.5 \text{ cm}^2$

The factor 3.556 cm has four significant figures, and the factor 2.4 cm has two significant figures. Therefore the answer must have two significant figures. The mathematical answer of 8.5344 cm<sup>2</sup> must be rounded back to 8.5 cm<sup>2</sup> in order for the answer to have two significant figures.

**Example:**  $(20.0 \text{ cm})(5.0000 \text{ cm}) = 100.00000 \text{ cm}^2 = 100. \text{ cm}^2$

The factor 20.0 cm has three significant figures, and the factor 5.0000 cm has five significant figures. The answer must be rounded to three significant figures. Therefore, the decimal must be written in to show that the two ending zeros are significant. If the decimal is omitted (left as an understood decimal), the two zeros will not be significant and the answer will be wrong.

**Example:**  $(5.444 \text{ cm})(22 \text{ cm}) = 119.768 \text{ cm}^2 = 120 \text{ cm}^2$

In this case, the answer must be rounded back to two significant figures. We cannot have a decimal after the zero in 120 cm<sup>2</sup> because that would indicate the zero is significant, whereas this answer must have exactly two significant figures.

## Summary

- Mathematics is a major tool for doing physics.
- The very large and very small measurements in physics make it useful to express numbers in exponential notation.
- It is also necessary in physics to do calculations with exponential numbers.
- There is uncertainty in all measurements.
- The use of significant figures is one way to keep track of uncertainty.
- Measurements must be written in the proper number of significant figures and the results of calculations must show the proper number of significant figures.
- Rules for Determining the Number of Significant Figures:



- All non-zero digits are significant.
- All zeros between non-zero digits are significant.
- All beginning zeros are *not* significant.
- Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal (the decimal point is not written in).

## Practice

Use these resources to answer the questions that follow.

The following url is for a video that teaches the concepts of exponential notation: <http://www.youtube.com/watch?v=ACZJMjt6qFk>.

The following url is for a video that teaches the concepts of significant figures: [http://www.youtube.com/watch?v=\\_jZAe9xeTFA](http://www.youtube.com/watch?v=_jZAe9xeTFA).

1. Why do we use scientific notation?
2. What is the purpose of significant figures?
3. Are leading zeros significant or not significant?"

## Review

1. Write the following numbers in proper exponential form.
  - (a) 3,120
  - (b) 0.00000341
2. Write the following numbers in expanded form.
  - (a)  $4.35 \times 10^6$
  - (b)  $6.1 \times 10^{-4}$
3. How many significant figures are in the following numbers?
  - (a) 2.3
  - (b) 17.95
  - (c)  $9.89 \times 10^3$
  - (d) 170
  - (e) 1.02
4. Perform the following calculations and give your answer with the correct number of significant figures:
  - (a)  $10.5 + 11.62$
  - (b)  $0.01223 + 1.01$
  - (c)  $19.85 - 0.0113$
5. Perform the following calculations and give your answer with the correct number of significant figures:
  - (a)  $0.1886 \times 12$
  - (b)  $2.995 \div 0.16685$
  - (c)  $910 \times 0.18945$

- **exponential notation:** A method of writing or displaying numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10. The scientific notation of 10,492, for example, is  $1.0492 \times 10^4$ .
- **significant digits:** The significant figures (also known as significant digits, and often shortened to sig figs) of a number are those digits that carry meaning contributing to its precision. This includes all digits *except*:
  - leading and trailing zeros which are merely placeholders to indicate the scale of the number.

- spurious digits introduced, for example, by calculations carried out to greater precision than that of the original data, or measurements reported to a greater precision than the equipment supports.

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## References

1. Serg C (Flickr: keko.). <http://www.flickr.com/photos/k3k0/3975819685/>. CC-BY 2.0
2. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0
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