Inertial Frames and Relative Motion

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CONCEPT 1 Inertial Frames and Relative Motion

The student will:

- explain frames of reference and inertial frames.
- solve problems involving relative motion in one dimension.
- solve problems involving relative motion in two dimensions.

Vocabulary

- **inertial frame:** A reference frame in which the observers are not subject to any accelerating force.
- **reference frame:** A coordinate system or set of axes within which to measure the position, orientation, and other properties of objects in it. It may also refer to an observational reference frame tied to the state of motion of an observer.
- **relative velocity:** The vector difference between the velocities of two bodies, or the velocity of a body with respect to another body which is at rest.

Equations

$\vec{V}_{ba} = \vec{V}_b - \vec{V}_a$ (velocity of b relative to a)

$\vec{V}_{ab} = \vec{V}_a - \vec{V}_b$ (velocity of a relative to b)

Introduction

Velocity is always measured relative to something. We measure how fast a person runs or how fast a car drives relative to the ground. However, we know from astronomy that the Earth itself is both turning around its axis and going around the Sun. A **reference frame** is a fixed point and we measure directions relative to it.

If you are on a bus going north at 60 mph, then the person seated across the aisle from you has velocity 60 mph north relative to the ground and velocity zero relative to you. If the bus is going at a steady speed, you can toss a coin across to them, and it works the same as if you were standing on solid ground. In the bus frame of reference, you and the other passenger have velocity zero, and the coin has a slight velocity east (say 20 mph). In this frame of reference, someone standing to the side of the road would have a velocity 60 mph south.

With the ground as your frame of reference, you and the other passenger are both moving 60 mph north, while the coin is moving diagonally northeast. The coin's velocity vector is 60 mph north and 20 mph east added together.

Both of these frames of reference are correct. You can solve any problem using either one, as long as you use it consistently. Some problems, though, are easier in one frame of reference than in another. If you wanted to solve how long it would take for the coin to go across the aisle, for example, then the bus frame of reference is much simpler.

Inertial Frames

There are endless examples of relative motion. Suppose that you're in an elevator that is rising with a constant speed of 2 m/s relative to the ground. If you release a ball while in this reference frame, how will the motion of the ball differ than had you dropped the ball while standing on the ground?

Had you been asleep in this reference frame and woke after the compartment was in motion, you would have no idea you were in motion. There is no experiment that can be performed to detect constant velocity motion. If you've ever traveled in a jet moving 1000 km/h (about 600 mph) with no air turbulence, then you know from firsthand experience that you felt motionless. After the brief acceleration period, you can no longer sense the motion of the elevator. The ball will move as if it has been released in the reference frame of Earth.

As a general statement, we consider all constant velocity reference frames to be equivalent. This idea is known as The Galilean Principle of Relativity. Constant-velocity reference frames are called **inertial frames** of reference. An "at-rest" reference frame is an arbitrary construct. If you're traveling with a constant velocity in your car, the reference frame of the car is an at-rest frame. The elevator compartment moving 2 m/s is an at-rest frame with respect to the compartment. You- who are sitting and reading this firmly placed on the earth- consider yourself to be in an at-rest reference frame. But you know the Earth itself is in motion. It rotates about its axis with a speed of about 1600 km/h (1000 mph) at the equator, and it orbits the sun with an average speed of 108,000 km/h (67,000 mph). In fact, the Earth isn't even an inertial frame of reference, since it rotates about its axis and its orbital speed varies. (Remember, velocity is constant only if its magnitude and direction do not change—objects in circular motion do not qualify!) We usually approximate the Earth as an inertial frame of reference since we do not readily sense the earth's acceleration. Objects on Earth's surface have a maximum acceleration due to its rotation of about 0.03 m/s^2—which we don't typically concern ourselves with since the acceleration due to gravity is 10 m/s^2.

Relative Motion: Part 1

The **Figure** below, a moving walkway, provides us with our first example of relative motion. Let us consider two Cartesian coordinate systems. One is attached to the "stationary" Earth. The other is attached to a walkway moving with a constant horizontal velocity of 1 m/s with respect to the earth. If a ball is thrown with an initial horizontal velocity of 3 m/s in the direction the walkway is moving by a person standing on the walkway, what horizontal velocity does a person standing on the ground measure for the ball? The person in the Earth frame sees the ball having a combined velocity of 4 m/s. The person in the "moving frame" will measure it as 3 m/s. According to The Principle of Galilean Relativity, the velocity, V, seen from the at-rest frame is additive, that is, $V = 1\ m/s + 3\ m/s$. See **Figure** below.

http://demonstrations.wolfram.com/RelativeMotionInASubwayStation/

Two cars are headed toward each other. Car$A$moves with a velocity of 30 mph due east and Car$B$with a velocity of 60 mph due west, relative to "at-rest" earth. See**Figure** below.

a. What is the velocity of car B relative to the velocity of car A?

We define motion to the east as positive (+30 mph), and motion to the west as negative (-60 mph).

From our previous statements regarding **relative velocity** we can "feel" that the relative velocity is greater than either speed: If we define the relative velocity (the velocity of car B relative to the velocity of car A) as: $\vec{V}_{ba} = \vec{V}_b - \vec{V}_a$, then $-60 - 30 = -90\ mph$, a person in car A sees car B moving west at 90 mph. The person in car A sees himself as "motionless" while car B is moving toward him with car B's speed and his own speed which he does not perceive.

b. What is the velocity of car A relative to the velocity of car B? This means we are assuming car B is our "at rest" coordinate system.

$\vec{V}_{ab} = \vec{V}_a - \vec{V}_b = 30$ mph $- (-60$ mph$) = +90$ mph. A person in car B sees car A moving east at 90 mph.

The person in car B sees himself as motionless, while car A is moving toward him with car A's speed and his own speed, which he does not perceive.

FIGURE 1.1

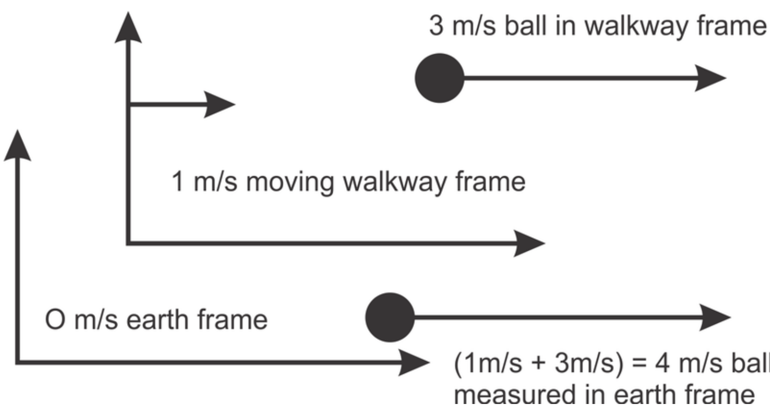

FIGURE 1.2

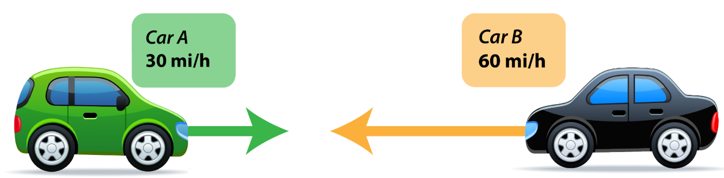

FIGURE 1.3

Part a: Passengers in the Car reference frame assume they are motionless. Part b: Passengers in the Car reference frame assume they are motionless.

Note: The "at-rest" frame sees its motion reversed in the "moving frame."

Car A is moving due east with a speed of 30 mph and Car B is moving due north with a speed of 30 mph.

a. What is the velocity of car A relative to car B?

$\vec{V}_{ab} = \vec{V}_a - \vec{V}_b$ The two vectors are not along the same line so we'll use their components

$\vec{V}_a = (+30, 0)$ and $\vec{V}_b = (0, +30)$, where east is $+x$ and north is $+y$

$-\vec{V}_b = (0, -30)$, therefore, $\vec{V}_a - \vec{V}_b = (30 + 0, 0 - 30) = (30, -30)$. The components are directed east and south, so

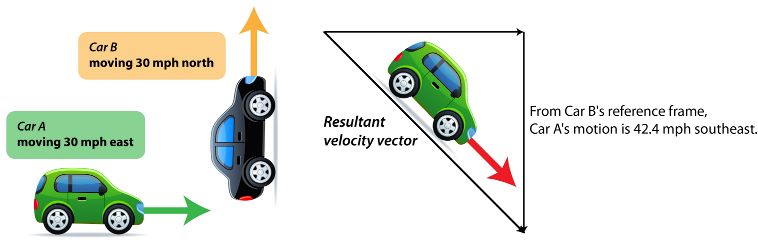

FIGURE 1.4
Diagram for 3a

the direction is southeast. Since both components have the same magnitude, the angle must be 45°. But since the vector is in the southeast direction it is in the 4^{th} quadrant so the angle is 315°, and the magnitude is the Pythagorean sum $(30^2+30^2)^{\frac{1}{2}} = 42.4\ mph$

Thus: $\vec{V}_{ab} = 42.4$ mph in direction 315°

b. What is the velocity of car B relative to car A?

The magnitude of the relative speed is the same, but the direction is reversed.

Thus: $\vec{V}_{ba} = 42.4$ mph in direction 135°

Relative Motion: Part 2

http://demonstrations.wolfram.com/ResultantOfAVector/

We begin Part 2 with a boat trip!

FIGURE 1.5
A boat crossing a calm body of water.

A boat moving at 4.0 m/s crosses a still lake, leaving from Point A and arriving at Point B. The distance between points A and B is 100 m.

The path of the boat is directly from A to B and the time of travel is quickly found: $\frac{100}{4} = 25\ s$. (See **Figure** below)

What if the situation was changed to a river with a current of 3.0 m/s flowing due east, while the boat still leaves from point A, attempting to head due north to reach point B, moving at 4.0 m/s relative to the water? If the person

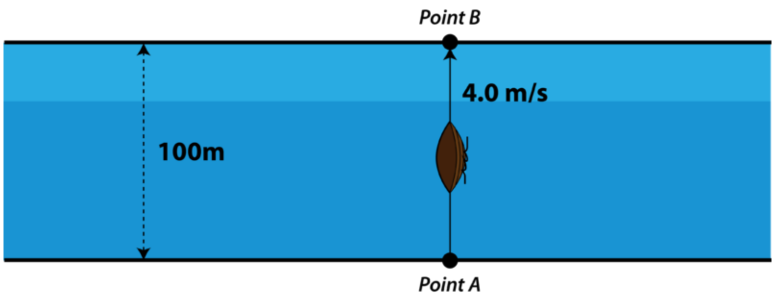

FIGURE 1.6

steering the boat does not take the current of the water into consideration, the boat will not reach point *B*. The boat will arrive somewhere downstream from point *B*. Let's analyze this situation further: (See **Figure** below)

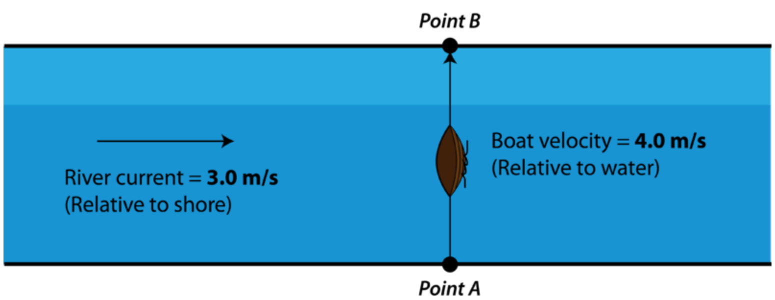

FIGURE 1.7

Assuming the boat is aimed due north with no attempt to compensate for the current:

1. How much time is required for the boat to reach the opposite bank of the river?
2. What distance downstream of Point *B* is the boat when it arrives at the opposite bank of the river?
3. What is the speed of the boat with respect to an observer at Point *A*?
4. What is the direction of the boat's motion with respect to an observer at Point *A*?

In order to answer the first question we must decide if the presence of the current increases or decreases the northerly speed of the boat.

What do you think?

Let's think this through.

If the current were flowing northeast or northwest, the boat's northerly speed would increase (think of a tail wind adding speed to an airplane's motion). We already agree that the eastern (or western) component will cause the boat to veer off course.

If the current, however, were flowing southeast or southwest, then the boat's northerly speed would decrease (think of a headwind, subtracting speed from an airplane's motion). Again, the eastern (or western) component of the current's motion will cause the boat to veer off course. (See **Figure** below)

We must conclude that a sideways (east or west) current cannot affect the forward (northward) motion of the boat. True, it can change the overall velocity of the boat. In **Figure** below the boat is moving both east, due to the current, *and* north under its own power. So, as seen from Point *A* or Point *B*, the boat is moving northeast. Since its velocity due north has not been affected by the current, the boat's northern component of speed in still 4.0 m/s, and since the boat moves east with the current, it now has an eastward velocity of 3.0 m/s.

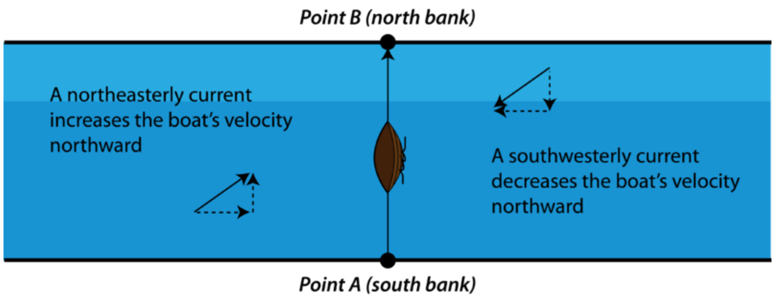

FIGURE 1.8

Answers to questions 1 – 4:

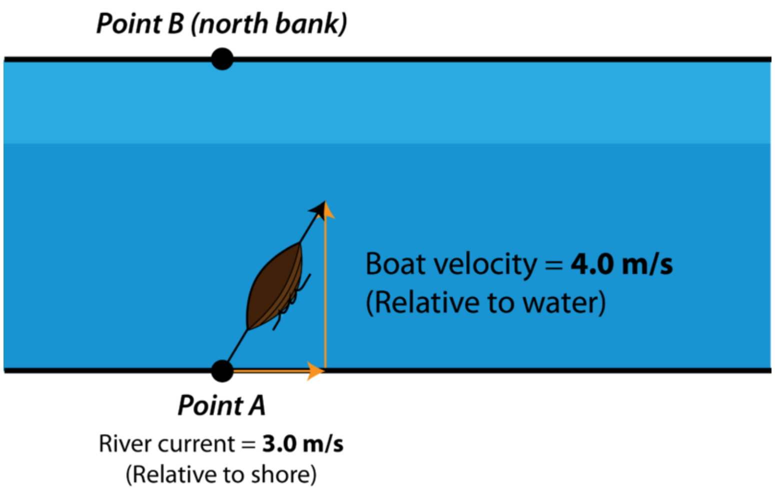

FIGURE 1.9

1. The current only acts perpendicular to the boat's motion, so the boat still moves 4.0 m/s due north. Hence, the time to cross remains $\frac{100}{4.0} = 25\ s$.
2. The boat has the same speed as the current so it travels 3.0 m/s for 25 s eastward. The boat therefore is $(3.0)(25) = 75\ m$ downstream from point B.
3. The boat is moving 4.0 m/s north and 3.0 m/s east. Its resultant speed can be found using the Pythagorean formula: square root of $\sqrt{3^2+4^2} = 5\ m/s$.
4. The angle as measured from a north-south line (line AB) can be determined using any of the trigonometric functions, sine, cosine, or tangent, since all three sides of the right triangle are known. For example, using the tangent function we have: $\tan^{-1}\left(\frac{3}{4}\right) = 36.87°$.

A final consideration is the interesting case of determining how to steer the boat such that it follows a straight course from point A to point B, in spite of the current. Recall that if there were no current the boat could just be aimed due north and it would easily move from point A to point B. We can intuit that the only hope of having the boat go from point A to point B is to aim the boat, somewhat, "into the current", or upstream. In other words, we can effectively travel straight across as we did in the "no-current" situation by aiming the boat some amount into the current. Why does this work?

First, consider what happens if a boat is aimed directly upstream with a speed equal to that of the current. The boat has a velocity of 3.0 m/s west and the water current has a velocity of 3.0 m/s east. What would someone positioned at point A see the boat do? If you think of the motions as two vectors for a moment: +3.0 m/s (current) + -3.0 m/s (boat), the vector sum is zero. In other words, the boat appears to be motionless to the observer at point A. So if the boat in our problem is aimed into the current such that its westward component is 3.0 m/s, we effectively have a situation equivalent to no current. The boat has a velocity of 4.0 m/s relative to the water and its westward component must be 3.0 m/s relative to the water if it is to cross directly from A to B. Using the Pythagorean formula we find the boat's northern component of motion to be the square root of 7.0 m/s or about 2.65 m/s. The boat will move due north at 2.65 m/s and arrive at the point B in $\frac{100}{2.65} = 37.7\ seconds$. With no current, the trip took 25 seconds. Some of the boat's forward motion had to be sacrificed in order to maintain the correct direction of travel. We can also enquire about the direction the boat was aimed in order to make the trip directly to point B. Since the components of the vector triangle are known, any trigonometric function will do. Let's use the sine and find the angle as measured from the north-south line.

$\sin^{-1}(\frac{3}{4}) = 48.59°$

Note, this was not the same angle that resulted when the boat was aimed due north and the current carried it downstream of B.

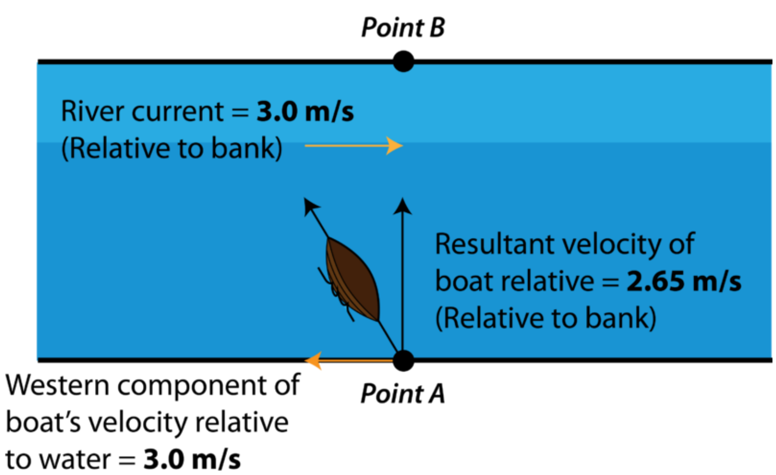

FIGURE 1.10

References

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