Independence of Motion Along Each Dimension

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the **FlexBook**®, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the **FlexBook Platform**®.

Printed: August 22, 2013

flexbook
next generation textbooks

BY NC SA

CONCEPT 1 Independence of Motion Along Each Dimension

The student will:

- understand how motion along each axis can be resolved independently
- solve problems involving objects which are simultaneously under the influence of uniform acceleration and constant velocity along different dimensions

Vocabulary

- **free fall:** The condition of acceleration due only to gravity. An object in free fall is not being held up, pushed, or pulled by anything except its own weight. Though objects moving in air experience some force from air resistance, this is sometimes small enough that it can be ignored and the object is considered to be in free fall.

- **instantaneous velocity of a projectile:** The velocity of an object at one instant during its motion. In the case of a projectile, the instantaneous velocity vector would be the result of a constant velocity horizontal motion and an accelerated velocity vertical motion.

- **projectile motion:** Projectile motion is a form of motion where a projectile is thrown near the Earth's surface with some horizontal component to its velocity. The projectile moves along a curved path under the action of gravity. The path followed by a projectile is called its trajectory. Projectile motion is motion in two directions. In the vertical direction, the motion is accelerated motion and in the horizontal direction, the motion is constant velocity motion.

- **range:** A projectile launched with specific initial conditions will travel a predictable horizontal displacement before striking the ground. This distance is referred to as the projectile's range.

Equations

$x_f = v_x t + x_i$

$y_f = \frac{1}{2}gt^2 + y_i$ (only for no initial vertical velocity)

Independence of Motion

The key to understanding motion in two or more dimensions is one principle: *Motion in each dimension works independently.*

What does this mean?

If we slide an object along a horizontal surface with little friction, like a hockey puck over ice, it will keep going in the same direction and speed - constant velocity. If we drop an object in the air, it will fall with speed increasing at the same rate - constant acceleration. What happens if we combine these, like if we slide an object off the end of a table so that it falls?

- In the horizontal direction, it continues with the constant speed.
- In the vertical direction, it speeds up with constant acceleration exactly as if dropped.

These two combine to make a path (or trajectory) that curves downward. This is a special case of what is called **projectile motion**.

Free Fall

Free fall is an idealized state of motion in which air resistance is neglected and only gravity acts upon a falling object. We consider situations in which objects are in free fall, after being launched with a certain horizontal velocity. Examples of such motion would be a baseball thrown with an initial horizontal velocity (a "line-drive") or rifle aimed horizontally and shot. We will discuss the more general case of an object fired at some angle above the horizontal in the next section.

Take two pennies and place them on a table top. Position one penny close to the edge of the table. Then, slide the other penny into the stationary penny with a glancing (very off center) blow. Both pennies will fall off of the table, but the one that was motionless will fall almost straight down (you may have to try this several times) while the other penny should slide off the table with an evident horizontal velocity. Have a friend watch the result to confirm that the pennies impact the ground at about the same instant. Gravity does not care how fast an object moves horizontally (just as the forward motion of the boat, in the last section, was unaffected by the current.)

The faster-moving penny follows an obvious parabolic path to the ground. The parabola is the result of the penny engaged in two one-dimensional motions, simultaneously: horizontal motion at a constant velocity (air resistance is negligible) and vertically accelerated motion due to gravity. Because the penny accelerates vertically, it does not move equal distances in equal times, as is the case for the horizontal direction. If the horizontal and vertical distances stayed in a fixed ratio to each other, the penny would appear to fall diagonally, not with the curve of a parabola.

http://groups.physics.umn.edu/demo/mechanics/movies/1D6020.mov

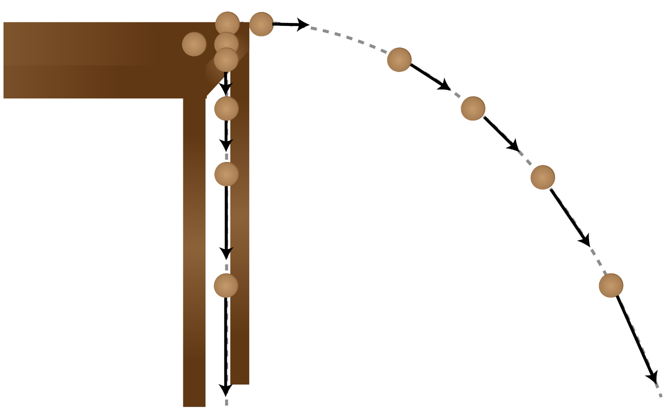

FIGURE 1.1

Check Your Understanding

A gun is positioned horizontally 1.75 m above the ground. A bullet exits with a muzzle velocity of 400 m/s. At the same instant as the bullet leaves the gun, an identical bullet is dropped from the height of the gun barrel. The second bullet falls straight down, landing at the feet of the shooter.

1. Do the bullets hit the ground at the same time?

Answer: Yes, they do.

2. What is the **range** of the fired bullet? By "range" we mean the horizontal distance the bullet has traveled.

Answer: The answer to this question relies on two pieces of information: (1) the horizontal velocity of the bullet and (2) the time it takes the bullet to fall to the ground. Since we're told how fast the bullet is moving horizontally (400 m/s) we need to know how much time elapses before bullet hits the ground. Once we have the time, the range

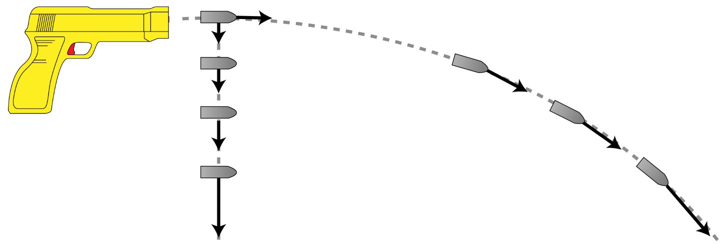

FIGURE 1.2

is only a matter of using $x = vt$, where x is the range of the bullet. The time is found by using the one-dimensional kinematic equation:

$$y_f = \frac{1}{2}at^2 + v_i t + y_i$$

Setting $y_f = 0, a = g = -10 \text{ m/s}^2, v_i = 0$ (in the $y-$direction), $y_i = 1.75\text{m}$

Therefore, $0 = \frac{1}{2}(-9.8)t^2 + 1.75, t = 0.598\ s$.

So, the range of the bullet is: $x = vt = (400\text{m/s})(0.589\text{s}) = 239.2 = 239\text{m}$.

By convention we often specify x as R for the range, hence $R = 239\ m$.

Note, this is an idealized example since air resistance is substantial for a bullet traveling at a speed of 400 m/s.

3. Based on the above idealized problem, we can see that the range of a gun fired horizontally is dependent upon:

a. The height at which the gun is fired.
b. The muzzle velocity of the bullet from the barrel.
c. Both the height and the muzzle velocity
d. Both a and b and the weight of the bullet.

Answer: The correct answer is C. Consider a bullet fired from the same height but an exit speed of 500 m/s:

range $= 500(0.598) = 299\ m$.

Consider the height of the gun lowered to 1.50 m with the same exit speed of 400 m/s.

Time to ground and time of flight = 0.553 s, $r = 400(0.553) = 221.3\ m$.

Here is a video demonstrating this experiment:

http://www.youtube.com/watch?v=D9wQVIEdKh8

Lab Example

Students are given the task of finding the muzzle velocity for a toy dart gun. The gun is held horizontally and the distance from the floor to the dart is 1.5 meters. The dart is fired half a dozen times and the average horizontal displacement is 6.0 meters. What is the velocity of the dart as it exits the barrel of the gun? (See **Figure** below)

This motion has two components: Vertical free-fall and horizontal motion at constant velocity. A chart to keep track of the data is useful.

TABLE 1.1:

Horizontal	Vertical
$x = 6.0\ m$	$y = 1.5\ m$

TABLE 1.1: (continued)

Horizontal	**Vertical**
$v = ?$	$v = 0\ m/s$
$a = 0$	$a = -10\ \text{m/s}^2$
$t = ?$	$t = ?$

The amount of time that the dart takes to hit the ground is also the amount of time it spends traveling horizontally. Therefore, once the time for the dart to fall 1.5 m is calculated, it can be used to determine the dart's horizontal velocity. Furthermore, since we can determine the vertical velocity of the projectile at any point along its trajectory, we can also determine its **instantaneous velocity** at any time during its flight.

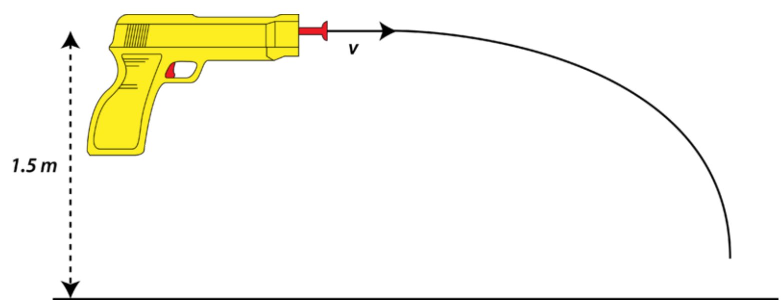

FIGURE 1.3

Using kinematics in one dimension, we find the time for the dart to hit the ground: $y_f = \frac{1}{2}at^2 + v_it + y_iy_f = 0, y_i = 1.5\ m$ and $a = g = -10\ \text{m/s}^2$.

Inserting our values, we find the time equals 0.55 seconds. Therefore, the dart has traveled a horizontal distance of 6.0 m in 0.55 seconds. Its horizontal component of velocity is therefore

$$v_x = \frac{6.0\text{m}}{0.55\text{s}} = 10.9\ \text{m/s}$$

When it hits the floor after 0.55 seconds, it has the same horizontal velocity that it started with. Its vertical velocity at that time is

$v_y = at = (-10\text{m/s}^2)(0.55\text{s}) = -5.5\text{m/s}$

The vertical and horizontal velocities are independent, and can be solved separately.

Plotting the Motion of the Dart in the x and y Directions

It would be instructive to graphically display the horizontal and vertical motions of the dart since their graphical forms are different. The horizontal motion of the dart has constant velocity. The dart covers equal horizontal displacements in equal time, and its representation in a position-time graph is linear as seen in **Figure** below.

The vertical motion of the dart has constant acceleration. The dart does not cover equal vertical displacements in equal time, and its representation in a position-time graph is parabolic as seen in **Figure** below.

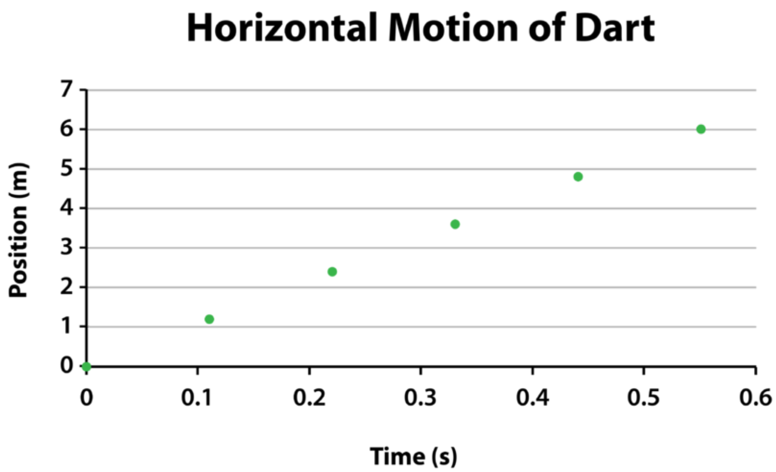

FIGURE 1.4
The horizontal motion of the dart has constant velocity.

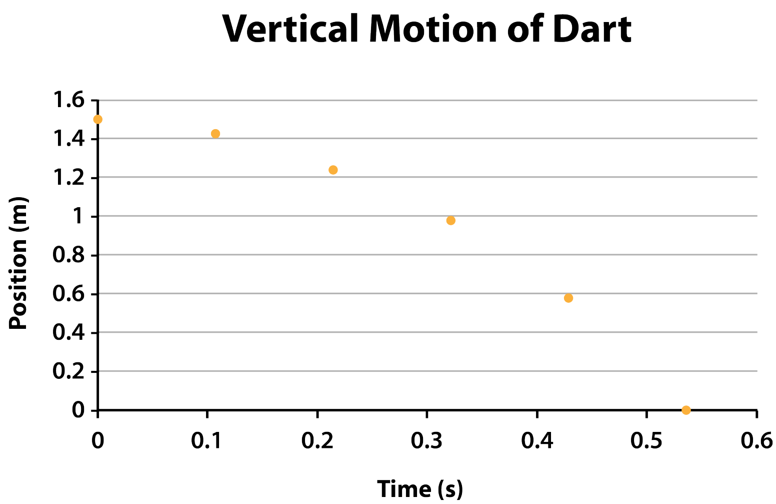

FIGURE 1.5
The vertical motion of the dart undergoes acceleration.

References

1. CK-12 Foundation - Raymond Chou. . CC-BY-NC-SA 3.0
2. CK-12 Foundation - Christopher Auyueng, Raymond Chou. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Christopher Auyueng. . CC-BY-NC-SA 3.0
4. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0
5. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0