## Graphing

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## Graphing

## Lesson Objectives

The student will:

- correctly graph data with the proper scale, units, and best fit curve.
- recognize patterns in data from a graph.
- solve for the slope of given line graphs.


## Vocabulary

extrapolation the process of creating data points beyond the end of a line graph, using the basic shape of the curve as a guide
graph a pictorial representation of the relationship between variables on a coordinate system
interpolation the process of estimating values between measured values
slope the ratio of the change in one variable with respect to the other variable

## Introduction

Scientists search for regularities and trends in data. To make it easier to find these regularities and trends, scientists often present data in either a table or a graph. The table below presents data about the pressure and volume of a sample of gas. You should note that all tables have a title and include the units of the measurements. The unit of pressure used here is atm (atmosphere).

## Data Table A: Pressure vs. Volume Data for a Gas Sample

| Pressure (in atm) | Volume (in liters) |
| :--- | :--- |
| 0.50 | 10.0 |
| 1.00 | 5.00 |
| 1.50 | 3.33 |
| 2.00 | 2.50 |
| 2.50 | 2.00 |
| 3.00 | 1.67 |

You may note a regularity that appears in this table: as the pressure of the gas increases, its volume decreases. This regularity or trend becomes even more apparent in a graph of this data. A graph is a pictorial representation of the relationship between variables on a coordinate system.


When the data from Data Table A is plotted as a graph, the trend in the relationship between the pressure and volume of a gas sample becomes more apparent. The graph aids the scientist in the search for any regularity that may exist in the data.

## Drawing Line Graphs

Reading information from a line graph is easier and more accurate as the size of the graph increases. In the example below, the graph on the left uses only a small fraction of the space available on the graph paper. The graph on the right shows the same data but uses all the space available. If you were attempting to determine the pressure at a temperature of 110 K , using the graph on the left would give a less accurate result than using the graph on the right.


When you draw a line graph, you should arrange the numbers on the axes to use as much of the graph paper as you can. If the lowest temperature in your data is 100 K and the highest temperature in your data is 160 K , you should arrange for 100 K to be on the extreme left of your graph and 160 K to be on the extreme right of your graph. The creator of the graph on the left did not take this advice and did not produce a very good graph. You should also make sure that the axes on your graph are labeled and that your graph has a title.

## Reading Information from a Graph

When we draw a line graph from a set of data points, we are inferring a trend and constructing new data points between known data points. This process is called interpolation. Even though we may only have a few data points, we are estimating the values between measured points, assuming that the line connecting these data points is a good model of what we're studying.

Consider the following set of data for the solubility of $\mathrm{KClO}_{3}$ in water. Data Table B shows that there are exactly six measured data points. When the data is graphed, however, the graph maker assumes that the relationship between the temperature and the solubility exists for all points within the data range. The graph maker draws a line by interpolating the data points between the actual data points. Note that the line is not drawn by just connecting the data points in a connect-the-dot manner. Instead, the line is a smooth curve that reasonably connects the known data points.

Data Table B: Solubility of Potassium Chlorate

| Temperature $\left({ }^{\circ} \mathbf{C}\right)$ | Solubility $\left(\mathbf{g} / \mathbf{1 0 0} \mathbf{~ m L ~} \mathbf{H}_{\mathbf{2}} \mathbf{O}\right)$ |
| :--- | :--- |
| 0 | 3.3 |
| 20 | 7.3 |
| 40 | 13.9 |
| 60 | 23.8 |
| 80 | 37.5 |
| 100 | 56.3 |

We can now read Graph B1, shown below, for points that were not actually measured. If we wish to determine the solubility of $\mathrm{KClO}_{3}$ at $70^{\circ} \mathrm{C}$, we follow the vertical grid line for $70^{\circ} \mathrm{C}$ up to where it touches the graphed line and then follow the horizontal grid line to the axis to read the solubility. In this case, we would read the solubility to be $30.0 \mathrm{~g} / 100 \mathrm{~mL}$ of $\mathrm{H}_{2} \mathrm{O}$ at $70^{\circ} \mathrm{C}$.

Graph B1: Solubility of Potassium Chlorate


There are also occasions when scientists wish to know more about points that are outside the range of measured data points. Extending the line graph beyond the ends of the original line, using the basic shape of the curve as a guide, is called extrapolation.


Suppose the graph for the solubility of potassium chlorate has been made from just three measured data points. If the actual data points for the curve were the solubility at $60^{\circ} \mathrm{C}, 80^{\circ} \mathrm{C}$, and $100^{\circ} \mathrm{C}$, the graph would be the solid line shown in Graph B2 above. If the solubility at $30^{\circ} \mathrm{C}$ was desired, we could extrapolate the curve (the dotted line) and obtain a solubility of $5.0 \mathrm{~g} / 100 \mathrm{~mL}$ of $\mathrm{H}_{2} \mathrm{O}$. If we check the more complete graph above (Graph B1), you can see that the solubility at $30^{\circ} \mathrm{C}$ is closer to $10 . \mathrm{g} / 100 \mathrm{~mL}$ of $\mathrm{H}_{2} \mathrm{O}$. The reason the second graph produces such a different answer is because the real behavior of potassium chlorate in water is more complicated than the behavior suggested by the extrapolated line. For this reason, extrapolation is only acceptable for graphs where there is evidence that the relationship shown in the graph will hold true beyond the ends of the graph. Extrapolation is more dangerous that interpolation in terms of producing possibly incorrect data.

In situations where it is unreasonable to interpolate or extrapolate data points from the actual measured data points, a line graph should not be used. If it is desirable to present data in a graphic form but a line graph is not useful, a bar graph can often be used instead. Consider the data in the following table.

Data Table C:Trout Creek Rainfall 1980-1989

| Year | Rainfall (in inches) |
| :--- | :--- |
| 1980 | 24.7 |
| 1981 | 21.2 |
| 1982 | 14.5 |
| 1983 | 13.2 |
| 1984 | 12.1 |
| 1985 | 16.8 |
| 1986 | 19.9 |
| 1987 | 29.2 |
| 1988 | 31.6 |
| 1989 | 21.0 |

For this set of data, you would not plot the data on a line graph because interpolating between years does not make sense; the concept of the average yearly rainfall halfway between the years 1980 and 1981 would not make sense. Looking at the general trend exhibited by Data Table C also does not provide the slightest amount of evidence about the rainfall in 1979 or 1990. Therefore, the interpolation and extrapolation of the data in this table is not reasonable. If we wish to present this information in a graphic form, a bar graph like the one seen in Graph C would be best.

Graph C:Trout Creek Rainfall 1980-1989


From this bar graph, you could very quickly answer questions like, "Which year was most likely a drought year for Trout Creek?" and "Which year was Trout Creek most likely to have suffered from a flood?"

## Finding the Slope of a Graph

As you may recall from algebra, the slope of the line may be determined from the graph. The slope represents the rate at which one variable is changing with respect to the other variable. For a straight-line graph, the slope is constant for the entire line, but for a non-linear graph, the slope varies at different points along the line. For a straight-line graph, the slope for all points along the line can be determined from any section of the graph. Consider the following data table and the linear graph that follows.

## Data Table D:Temperature vs.Volume Data for a Gas Sample

| Temperature ( ${ }^{\circ} \mathbf{C}$ ) | Volume of Gas (mL) |
| :--- | :--- |
| 20 | 60 |
| 40 | 65 |
| 60 | 70 |
| 80 | 75 |
| 100 | 80 |
| 120 | 85 |



The relationship in this set of data is linear; in other words, the data produces a straight-line graph. The slope of this line is constant at all points on the line. The slope of a line is defined as the rise (change in vertical position) divided by the run (change in horizontal position). For a pair of data points, the coordinates of the points are identified as $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$. In this case, the data points selected are $\left(40^{\circ} \mathrm{C}, 65 \mathrm{~mL}\right)$ and $\left(100^{\circ} \mathrm{C}, 80 \mathrm{~mL}\right)$. The slope can then be calculated in the following manner:

$$
\text { slope } m=\frac{r i s e}{r u n}=\frac{\left(y_{2}-y_{1}\right)}{\left(x_{2}-x_{1}\right)}=\frac{(80 \mathrm{~mL}-65 \mathrm{~mL})}{\left(100^{\circ} \mathrm{C}-40^{\circ} \mathrm{C}\right)}=0.25 \mathrm{~mL} /{ }^{\circ} \mathrm{C}
$$

Therefore, the slope of the line is $0.25 \mathrm{~mL} /{ }^{\circ} \mathrm{C}$. The fact that the slope is positive indicates that the line is rising as it moves from left to right and that the volume increases by 0.25 mL for each $1^{\circ} \mathrm{C}$ increase in temperature. A negative slope would indicate that the line was falling as it moves from left to right.
For a non-linear graph, the slope must be calculated for each point independently. Since the line will be a curve, the slope is calculated from the tangent to the curve at the point in question. Data Table E and Graph E are for a
reaction in which the concentration of one of the reactants, bromine, was measured against time. The concentration is expressed in moles/liter, which is symbolized by M.

Data Table E: Concentration of Bromine vs. Time

| Time (seconds) | Concentration of $\mathbf{B r}_{2}(\mathbf{M})$ |
| :--- | :--- |
| 1.0 | 0.063 |
| 2.0 | 0.045 |
| 3.0 | 0.037 |
| 4.0 | 0.032 |
| 5.0 | 0.028 |
| 6.0 | 0.026 |
| 7.0 | 0.024 |



In order to determine the slope at some point on a curved line, a tangent (approximate) is drawn in as a line that just touches the point in question. Once the tangent has been drawn, the slope of the tangent is determined, which is also the slope of the curve at that point. In the graph above, the tangent has been drawn at the point where $t=2$ seconds. We determine the $x$ - and $y$-coordinates for two points along the tangent line (as best we can) and use the coordinates of those two points to calculate the slope of the tangent. The coordinates of the point at the left end of the tangent line is determined to be $(1.00 \mathrm{~s}, 0.056 \mathrm{M})$. The coordinates of the point at the right end of the line is harder to determine, and we are guessing that the coordinates are ( $3.25 \mathrm{~s}, 0.031 \mathrm{M}$ ).

$$
\text { slope } m=\frac{r i s e}{r u n}=\frac{\left(y_{2}-y_{1}\right)}{\left(x_{2}-x_{1}\right)}=\frac{(0.031 \mathrm{M}-0.056 \mathrm{M})}{(3.25 \mathrm{~S}-1.00 \mathrm{~S})}=-0.011 \mathrm{M} / \mathrm{s}
$$

Since the slope is a negative number, we know the line is decreasing in height. At $\mathrm{t}=2$ seconds, the concentration of bromine is decreasing at a rate of 0.011 moles/liter per second. At other points along this curve, the slope would be different. From the appearance of the curve, it is apparent that the slope is negative (the concentration of bromine is decreasing) all along the line, but it appears to be decreasing more quickly at the beginning of the reaction and less quickly as time increases.

## Lesson Summary

- Tables and graphs are two common methods of presenting data that aid in the search for regularities and trends within the data.
- When we draw a line graph from a set of data points, we are inferring a trend and constructing new data points between known data points. This process is called interpolation.
- Constructing data points beyond the end of a line graph, using the basic shape of the curve as a guide, is called extrapolation.
- The slope of a graph represents the rate at which one variable is changing with respect to the other variable.
- For a straight-line graph, the slope for all points along the line can be determined from any section of the graph.
- For a non-linear graph, the slope must be determined for each point by drawing a tangent line to the curve at the point in question.


## Further Reading / Supplemental Links

This website has lessons, worksheets, and quizzes on various high school chemistry topics. Lesson 16-5 is on graphing. The lesson provides some tips and some sample data to practice graphing with.

- http://www.fordhamprep.org/gcurran/sho/sho/lessons/lesson165.htm


## Review Questions

1. What would you do to find the slope of a curved line?
2. Andrew was completing his density lab for his chemistry lab exam. He collected the following data in his data table (shown in Table 1.1).
a. Draw a graph to represent the data.
b. Calculate the slope.
c. What does the slope of the line represent?

## TAble 1.1: Data Table for Problem 2

## Mass of Solid (g)

3.4
6.8
10.2
21.55
32.89
44.23
55.57

## Volume of Solution (mL)

0.3
0.6
0.9
1.9
2.9
3.9
4.9
3. Donna is completing the last step in her experiment to find the effect of the concentration of ammonia on the reaction. She has collected the following data from her time trials and is ready for the analysis. Her data table is Table 1.2. Help Donna by graphing the data, describing the relationship, finding the slope, and then discussing the meaning of the slope.

## TAble 1.2: $\quad$ Data Table for Problem 3

| Time (s) | Concentration (mol/L) |
| :--- | :--- |
| 0.20 | 49.92 |
| 0.40 | 39.80 |
| 0.60 | 29.67 |
| 0.81 | 20.43 |
| 1.08 | 14.39 |
| 1.30 | 10.84 |
| 1.53 | 5.86 |
| 2.00 | 1.95 |
| 2.21 | 1.07 |
| 2.40 | 0.71 |
| 2.60 | 0.71 |

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