Forces in Translational Equilibrium

next generation textbooks

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Concept 1 Forces in Translational Equilibrium

Objective

The student will:

- Understand how to apply Newton's Second Law under equilibrium conditions in two dimensions.

Vocabulary

- **static equilibrium:** Objects in static equilibrium are motionless.

Introduction

FIGURE 1.1

There are many objects we do not want to see in motion. In the **Figure** 1.1, the mountain climbers want their ropes to keep them from moving downward. We construct buildings and bridges to be as motionless as possible. We want the acceleration (and velocity) of these objects to be zero. For an object to be in **static equilibrium** (that is, motionless) the right-hand side of Newton's Second Law, $\sum F = ma$, must be zero. Thus, $\sum F = 0$. This equation is simple enough when an object is held with a single support. In an earlier example, we depicted Joe Loose hanging by a single rope. Joe's goal was to remain hanging in equilibrium (just like the climbers in the photograph). The force of gravity pulling Joe down was exactly balanced by the tension in the rope that supported him.

But Joe won't be hanging for very long, will he? You can see that the rope is slowly fraying against the mountainside (recall the original problem). Soon it will snap. But Joe's in luck, because a rescue team has come to his aid. They arrive just in time to secure two more ropes to the mountain side and toss Joe the slack to tie around his waist before his rope snaps! Joe is saved. But does Joe thank the rescue team like any sane person would? No. Instead, still in midair, he pulls out a pad and pencil from his back pocket in order to analyze the forces acting on him (**Figure** below).

FIGURE 1.2

Static Equilibrium—Saving Joe

In order for Joe to remain in equilibrium, he must not move in the $x-$ or $y-$directions. This means that the sum of all forces in the $x-$direction must add to zero. And the sum of all forces in the $y-$direction must add to zero.

The procedure for solving problems with forces in equilibrium is as follows:

1. Place **Figure** above in a coordinate plane with the object at the origin.
2. Resolve the tension vectors T_1 and T_2 into their $x-$ and $y-$components.
3. Use Newton's Second Law: $\sum F_x = 0$ and $\sum F_y = 0$.

In order to solve this problem, we'll need more information, including the angles that the ropes make with the vertical. The information is provided below, along with **Figure** below.

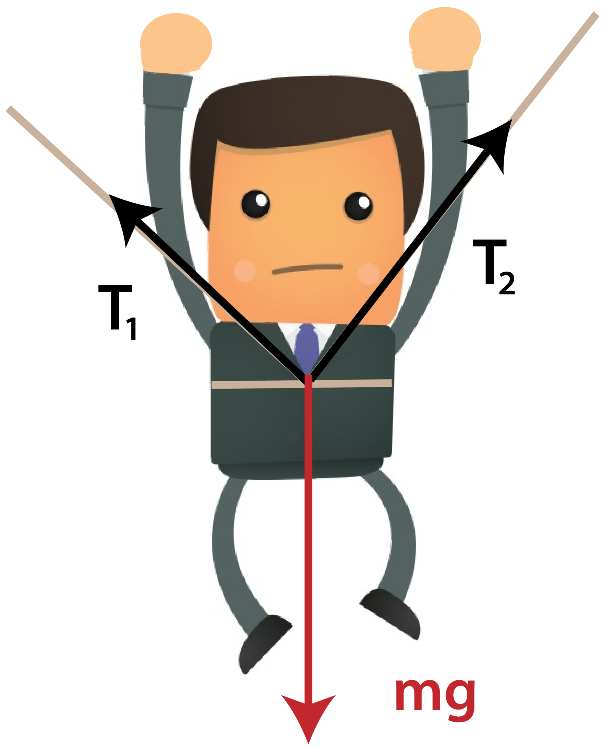

FIGURE 1.3

$\theta_1 = 45^\circ$

$\theta_2 = 30^\circ$

$Mg = 800\ N$

Find T_1 and T_2

The solution requires solving a set of simultaneous equations.

First, we find the components of vectors T_1 and T_2.

$$T_{1x} = T_1 \sin 45^\circ \textit{ and } T_{1y} = T_1 \cos 45^\circ$$
$$T_{2x} = T_2 \sin 30^\circ \textit{ and } T_{2y} = T_2 \cos 30^\circ$$

Next we apply Newton's Second Law.

$$\sum F_x : T_2 \sin 30^\circ - T_1 \sin 45^\circ = 0$$
$$\sum F_y : T_2 \cos 30^\circ - T_1 \cos 45^\circ = 800$$

The first equation can be quickly simplified to give $T_2 = \sqrt{2}T_1$. T_2 is then substituted in the second equation and T_1 is found. Once T_1 is found, T_2 can easily be computed using $T_2 = \sqrt{2}T_1$.

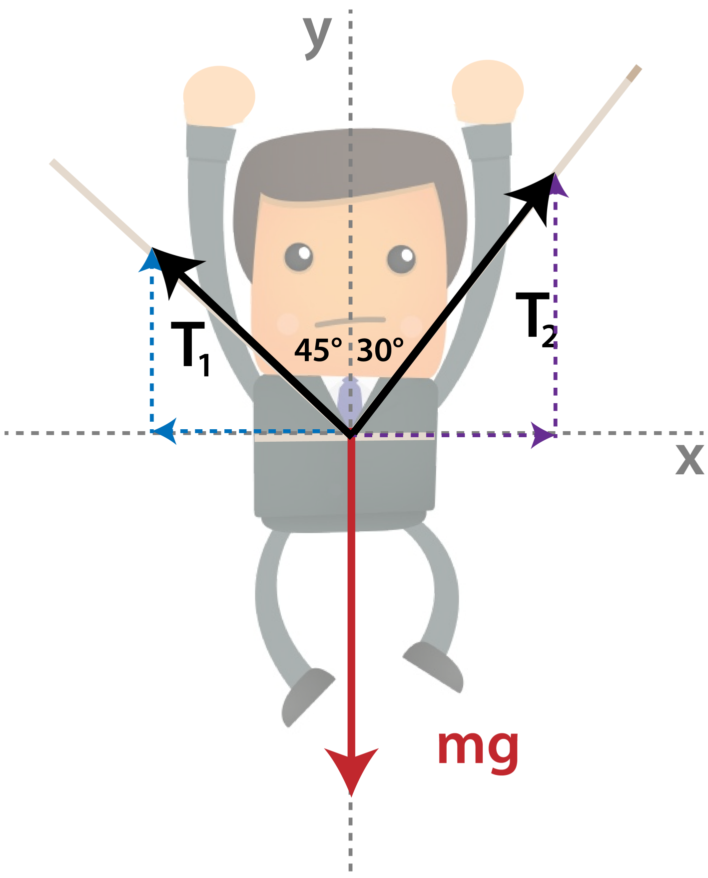

FIGURE 1.4

$$T_1 = 414.11 = 414 = 410\ N$$
$$T_2 = 585.56 = 586 = 590\ N$$

Check your understanding

What general equation can be written for $\sum_y F$ if the angles in **Figure** above are equal?

Answer: The sum of the $y-$components is responsible for supporting the weight. If the angles are equal, the ropes have equal tension. Therefore, the tension in the $y-$direction of either rope can quickly be found since:

$$\sum_y F = 2T_y = mg \rightarrow T_y = \frac{mg}{2}$$

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