Conservation of Momentum in Two Dimensions

Ck12 Science

AUTHOR
Ck12 Science

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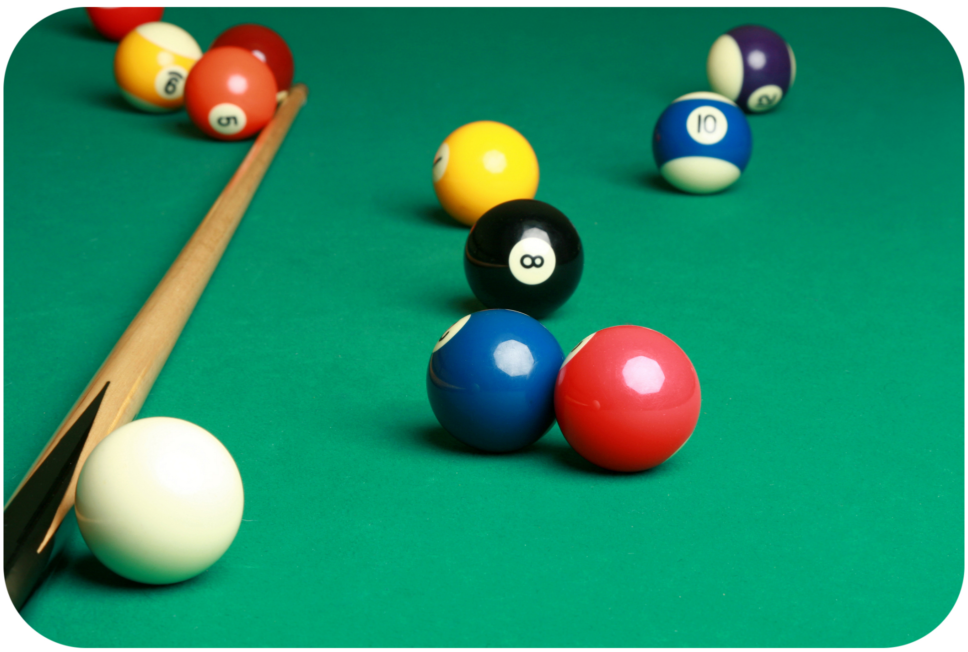
CONCEPT 1 Conservation of Momentum in Two Dimensions

- Use the conservation of momentum and vector analysis to solve two-dimensional collision problems.



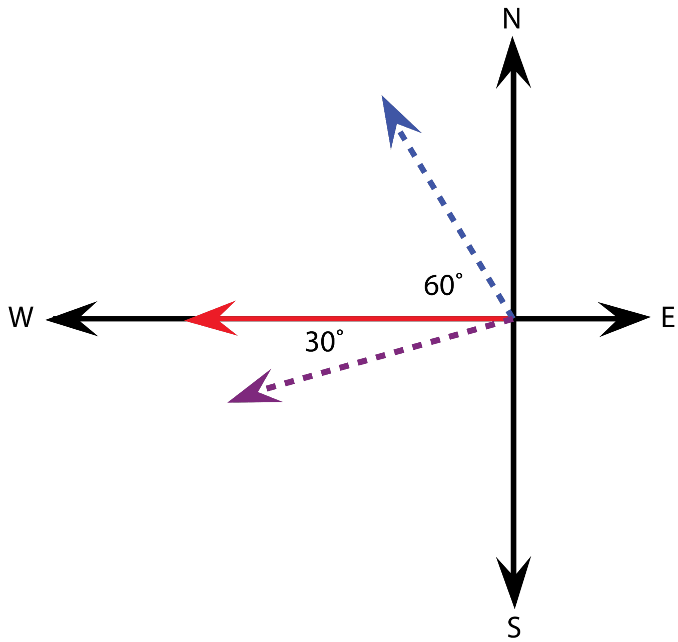
In a game of billiards, it is important to be able to visualize collisions in two dimensions –the best players not only know where the target ball is going but they also know where the cue ball is going.

Conservation of Momentum in Two Dimensions

The conservation of momentum law holds for all closed systems regardless of the directions of the particles before and after they collide. Momentum is a vector and therefore collisions of particles in two dimensions can be represented by axial vector components.

Example Problem: A 2.0 kg ball, *A*, is moving with a velocity of 5.00 m/s due west. It collides with a stationary ball, *B*, also with a mass of 2.0 kg. After the collision, ball *A* moves off at 30° south of west while ball *B* moves off at 60° north of west. Find the velocities of both balls after the collision.

Solution: Since ball *B* is stationary before the collision, then the total momentum before the collision is equal to momentum of ball *A*. The momentum of ball *A* before collision is $\rho = mv = (2.00 \text{ kg})(5.00 \text{ m/s}) = 10.0 \text{ kg m/s}$ west



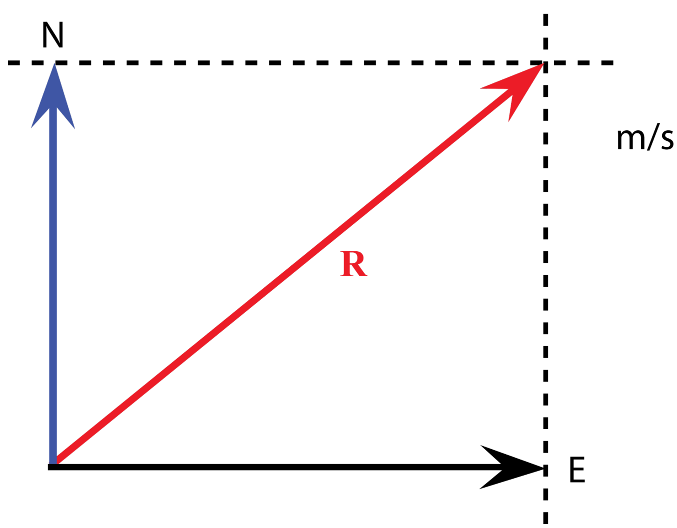
Since momentum is conserved in this collision, the sum of the momenta of balls *A* and *B* after collsion must be 10.0 kg m/s west.

$\rho_{A\text{after}} = (10.0 \text{ kg m/s})(\cos 30°) = (10.0 \text{ kg m/s})(0.866) = 8.66 \text{ kg m/s}$

$\rho_{B\text{after}} = (10.0 \text{ kg m/s})(\cos 60°) = (10.0 \text{ kg m/s})(0.500) = 5.00 \text{ kg m/s}$

To find the final velocities of the two balls, we divide the momentum of each by its mass, $v_A = 4.3$ m/s and $v_B =$ 2.5 m/s.

Example Problem: A 1325 kg car moving north at 27.0 m/s collides with a 2165 kg car moving east at 17.0 m/s. The two cars stick together after the collision. What is the speed and direction of the two cars after the collision?



Solution:

Northward momentum $= (1325 \text{ kg})(27.0 \text{ m/s}) = 35800 \text{ kg m/s}$

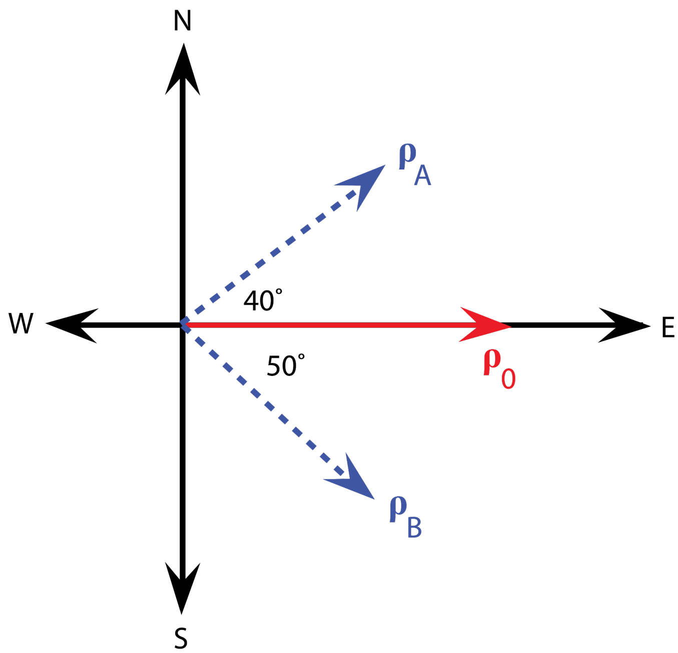
Eastward momentum $= (2165 \text{ kg})(17.0 \text{ m/s}) = 36800 \text{ kg m/s}$

$R = \sqrt{(35800)^2 + (36800)^2} = 51400 \text{ kg} \cdot \text{m/s}$

$\theta = \sin^{-1}\frac{35800}{51400} = 44^\circ$ north of east

$velocity = \frac{\rho}{m} = \frac{51400\text{ kg·m/s}}{3490\text{ kg}} = 14.7$ m/s @ 44° N of E

Example Problem: A 6.00 kg ball, *A*, moving at velocity 3.00 m/s due east collides with a 6.00 kg ball, *B*, at rest. After the collision, *A* moves off at 40.0° N of E and ball *B* moves off at 50.0° S of E.



a. What is the momentum of *A* after the collision?
b. What is the momentum of *B* after the collision?
c. What are the velocities of the two balls after the collision?

Solution: $\rho_{\text{initial}} = mv = (6.00\text{ kg})(3.00\text{ m/s}) = 18.0$ kg m/s

This is a right triangle in which the initial momentum is the hypotenuse and the two momenta after the collision are the legs of the triangle.

a. $\rho_A = (18.0\text{ kg m/s})(\cos 40.0^\circ) = (18.0\text{ kg m/s})(0.766) = 13.8$ kg m/s
b. $\rho_B = (18.0\text{ kg m/s})(\cos 50.0^\circ) = (18.0\text{ kg m/s})(0.643) = 11.6$ kg m/s
c. $v_A = 2.30$ m/s $\qquad v_B = 1.93$ m/s

Summary

- The conservation of momentum law holds for all closed systems regardless of the directions of the particles before and after they collide.
- Momentum is a vector and therefore collisions of particles in two dimensions can be represented by axial vector components.

Practice

This video shows circus performers using conservation of momentum. Use this resource to answer the questions that follow.

http://www.pbs.org/opb/circus/classroom/circus-physics/angular-momentum/

MEDIA

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1. Why do the fliers scrunch up in the air while spinning and twisting?
2. What happens to the rate at which they spin when they change shape in the air?

Review

1. Two bullets of identical mass are fired with identical speeds at identical blocks of wood sitting on ice. One of the bullets is made of rubber and bounces its wooden block. The other bullet is made of copper and penetrates and remains inside the other block of wood. Which block of wood will acquire the greatest velocity sliding on the ice? Why?
2. Billiard ball *A*, mass 0.17 kg, moving due east with a velocity of 4.0 m/s, strikes stationary billiard ball *B*, also mass of 0.17 kg. After the collision, ball *A* moves off at an angle of 45° north of east with a velocity of 2.0 m/s. What is the speed and direction of ball *B*?
3. A bomb, originally sitting at rest, explodes and during the explosion breaks into four pieces of exactly 0.25 kg each. One piece flies due south at 10 m/s while another pieces flies due north at 10 m/s.
   (a) What do we know about the directions of the other two pieces and how do we know it?
   (b) What do we know about the speeds of the other two pieces and how do we know it?
4. In a head-on collision between protons in a particle accelerator, three resultant particles were observed. All three of the resultant particles were moving to the right from the point of collision. The physicists conducting the experiment concluded there was at least one unseen particle moving to the left after the collision. Why did they conclude this?

References

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