Circular Motion

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CONCEPT 1 Circular Motion

Students will learn that in circular motion there is always an acceleration (and hence a force) that points to the center of the circle defined by the objects motion. This force changes the direction of the velocity vector of the object but not the speed. Students will also learn how to calculate that speed using the period of motion and the distance of its path (circumference of the circle it traces out).

Vocabulary

- **centripetal acceleration:** The inward acceleration that keeps an object in circular motion.
- **centripetal force:** The inward force that keeps an object in circular motion.

Introduction

A satellite orbits around the Earth in **Figure** below. A car travels around a curve in **Figure** below. All of these objects are engaged in circular motion. Let us consider the satellite first. The satellite is held in place by the Earth's gravity. The gravity holds the satellite in its orbit. In what direction does this force act? If the earth were "magically" gone, the satellite would fly off tangent to its motion at the instant gravity no longer held it. The force preventing this from happening must keep pulling the satellite toward the center of the circle to maintain circular motion.

FIGURE 1.1

What is the force in **Figure** above that prevents the car from skidding off the road? If you guessed "the friction between the tires and the road" you'd be correct. But is it static or kinetic friction? Unless the tires skid, there can be no kinetic friction. It is static friction that prevents the tires from skidding, just as it is static friction that permits you to walk without slipping. In **Figure** below, you can see the foot of a person who walks toward the right by pushing their foot backward with a horizontal component of force F. They move forward because the ground exerts a horizontal component force f_s in the opposite direction. (Note that vertical forces are ignored.) The force the ground exerts on the person's foot is a static friction force. Because the foot does not slide, we know that F and f_s are equal opposed forces. We can easily see which direction the static friction force must act when we walk,

FIGURE 1.2

but what about a car performing circular motion? In what direction does the static friction act on the car in **Figure** above?

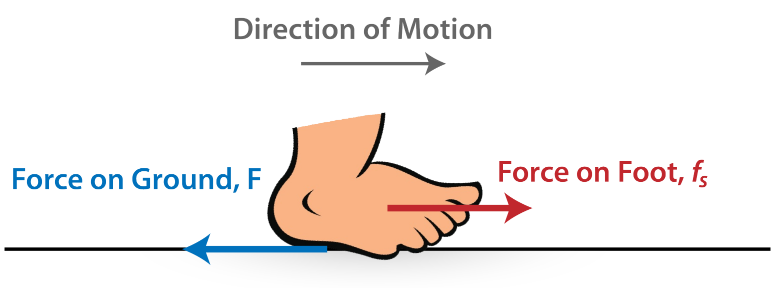

FIGURE 1.3

Figure below shows the top view of a car moving around a circular track with a constant speed. Since acceleration is defined as $a = \frac{\Delta v}{\Delta t}$, you may be tempted to say that since the speed remains constant, $\Delta v = 0$, the acceleration must also be zero. But that conclusion would be incorrect because Δv represents a change in velocity, not a change in speed. The velocity of the car is not constant since it is continuously changing its direction. How then do we find the acceleration of the car?

Figure below shows the instantaneous velocity vectors for the car in two different positions a very small time apart.

Notice that the vector ΔV points toward the center of the circle. (Recall that ΔV can be thought of as the sum of the vectors $V_2 + (-V_1)$.) The direction of the acceleration points in the direction of ΔV since acceleration is defined as $a = \frac{\Delta \vec{v}}{\Delta t}$. This is reasonable, since if there were no force directed toward the center of the circle, the car would move off tangent to the circle.

We call the inward force that keeps an object in circular motion a "center seeking", or **centripetal** force and the acceleration, centripetal acceleration. The centripetal acceleration is often denoted as a_c In order to find the correct expression for the magnitude of the centripetal acceleration we'll need to use a little geometric reasoning. **Figure** below and **Figure** below show two "almost" similar triangles.

The magnitudes of $-V_1$ and V_2 are equal, and the change in location of the car occurs over a very small increment in time, Δt. The velocities are tangent to the circle and therefore perpendicular to the radius of the circle. As such,

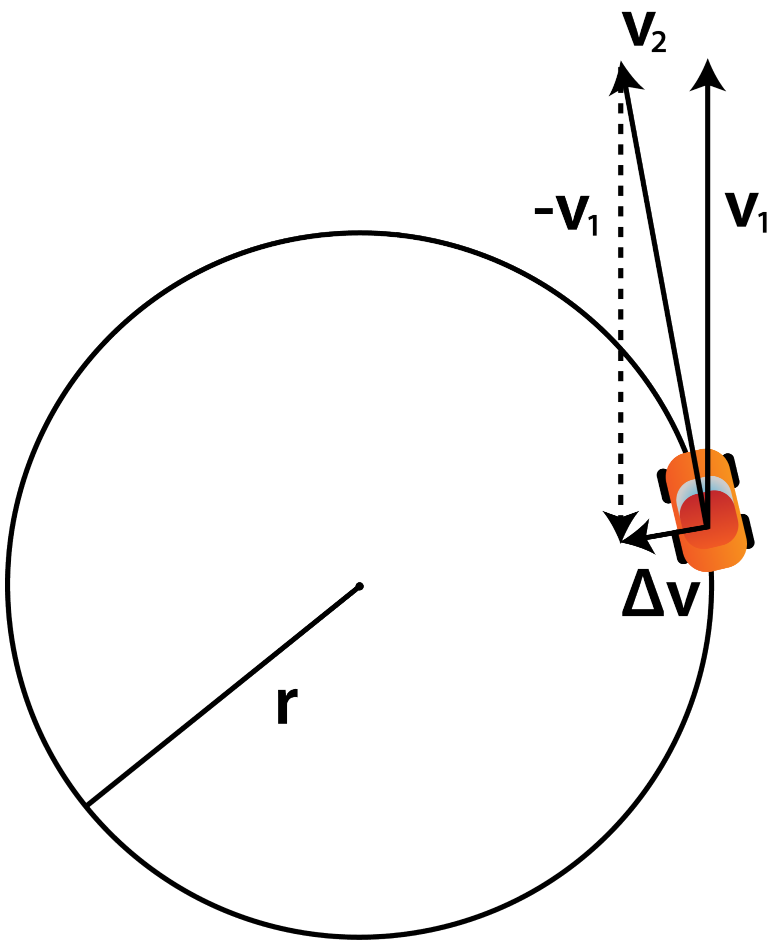

FIGURE 1.4

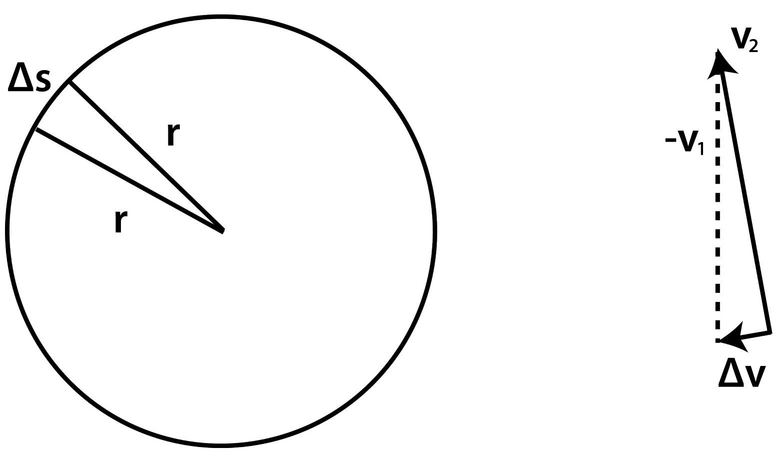

FIGURE 1.5

the "radius" triangle and the "velocity" triangle are approximately similar (see the figures above). We construct an approximate ratio between the two triangles by assuming that during the time Δt, the car has traveled a distance Δs along the circle. The ratio below is constructed in order to determine the acceleration.

$\frac{\Delta s}{r} \doteq \frac{\Delta v}{v}$, which leads to, $v\Delta s \doteq r\Delta v$.

Dividing both sides of the equation by Δt, we have: $v\frac{\Delta s}{\Delta t} \doteq r\frac{\Delta v}{\Delta t}$.

But $\frac{\Delta s}{\Delta t}$ is the speed v of the car and $\frac{\Delta v}{\Delta t}$ is the acceleration of the car.

If we allow the time to become infinitesimally small, then the approximation becomes exact and we have:$v^2 = ra, a = \frac{v^2}{r}$. Thus, the magnitude of the centripetal acceleration for an object moving with constant speed in circular motion is $a_c = \frac{v^2}{r}$, and its direction is toward the center of the circle.

Illustrative Examples using Centripetal Acceleration and Force

Example 1A: A 1000 kg car moves with a constant speed 13.0 m/s around a flat circular track of radius 40.0 m. What is the magnitude and direction of the centripetal acceleration?

Answer: The magnitude of the car's acceleration is $a_c = \frac{v^2}{r} = \frac{132}{40} = 4.225 = 4.23\ m/s^2$ and the direction of its acceleration is toward the center of the track.

Example 1b: Determine the force of static friction that acts upon the car in **Figure** below

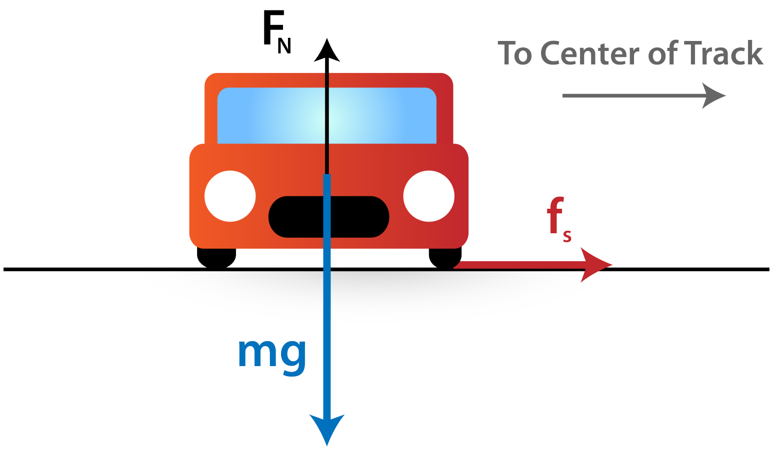

FIGURE 1.6

Answer:

Using Newton's Second Law: $\sum F = f_s = ma = 1000(4.225) = 4225 = 4230\ N$

Example 1c: Determine the minimum necessary coefficient of static friction between the tires and the road.

Answer:

$\sum_y F = F_N - Mg = 0, F_N = Mg$ but $f_s = \mu_s F_N = ma$

Thus, $\mu_s Mg = Ma, \mu_s g = a, \mu_s = \frac{a}{g} = \frac{4.225}{9.8} = 0.431 = 0.43$

Check Your Understanding

True or False?

1. Kinetic friction is responsible for the traction (friction) between the tires and the road.

Answer: False. As long as the car does not skid, there is no relative motion between the instantaneous contact area of the tire and the road.

2. True or False? The force of static friction upon an object can vary.

Answer: True. In attempting to move an object, a range of forces of different magnitudes can be applied until the maximum static friction between the object and the surface it rests upon is overcome and the object is set into motion. Recall that the magnitude of static friction is represented by the inequality: $f_s \leq \mu_s F_N$

3. The greater the mass of the car, the greater the coefficient of friction.

Answer: False. The coefficient of friction is independent of the mass of an object. Recall that it is the ratio of the friction force to the normal force. As such, it is a pure number dependent only upon the nature of the materials in contact with each other- in this case rubber and asphalt.

References

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