Basic Physics SE-Projectile Motion

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CHAPTER 1 Basic Physics SE-Projectile Motion

The Big Idea

In this chapter, we aim to understand and explain the parabolic motion of a thrown object, known as projectile motion. Motion in one direction is unrelated to motion in other perpendicular directions. Once the object has been thrown, the only acceleration is in the y (up/down) direction. The x (right/left) direction velocity remains unchanged.

Key Equations

In the vertical direction

- $\Delta y = v_{iy}t - \frac{1}{2}gt^2$
- $\Delta v_y = -gt$
- ${v_y}^2 = {v_{iy}}^2 - 2g(\Delta y)$
- $a_y = -g = -9.806\ m/s^2 \approx -10\ m/s^2$

In the horizontal direction

- $\Delta x = v_{ix}t$
- $v_x = v_{ix}$ (does not change)
- $a_x = 0$

A few comments on the above equations:

- Recall: The 'Δ' symbol means 'change in' so that for example $\Delta v_y = v_{fy} - v_{iy}$, which tells you the change in the velocity from the starting velocity to the final velocity.
- $+y$ direction is defined as upward for the above equations
- The initial velocity v_i can be separated into $v_{ix} = v_i \cos\theta$ and $v_{iy} = v_i \sin\theta$, where θ is the angle between the velocity vector and the horizontal.

Key Concepts

- In projectile motion, the horizontal displacement of an object is called its *range*.
- At the top of its flight, the vertical speed of an object is zero.
- To work these problems, use the equations above: one set for the $y-$direction (vertical direction), and one set for the $x-$direction (horizontal direction). The $x-$direction and $y-$direction don't "talk" to each other. They are separate dimensions. Keep them separate.

- The time is the same for the two directions, and can be plugged into both equations.
- Since in the absence of air resistance, there is no acceleration in the $x-$direction, the velocity in the $x-$direction does not change over time. This is a counter-intuitive notion for many. (Air resistance will cause the $x-$velocity to decrease slightly or significantly depending on the object. But this factor is ignored for the time being.)
- The $y-$direction motion must include the acceleration due to gravity, and therefore the velocity in the $y-$direction changes over time.
- The shape of the path of an object undergoing projectile motion is a parabola.
- We will ignore air resistance in this chapter. Air resistance will tend to shorten the range of the projectile motion by virtue of producing an acceleration opposite to the direction of motion.

Solved Examples

Example 1: A tennis ball is launched 32° above the horizontal at a speed of 7.0 m/s. What are the horizontal and vertical velocity components?

Question: v_x and $v_y = ?\ [m/s]$

Given: $v = 7.0\ m/s$

$\theta = 32^\circ$

Equation: $v_x = v\cos\theta \qquad v_y = v\sin\theta$

Plug n' Chug: $v_x = v\cos\theta = (7.0\ m/s)\cos(32^\circ) = 5.9\ m/s$

$v_y = v\sin\theta = (7.0\ m/s)\sin(32^\circ) = 3.7\ m/s$

Answer: $\boxed{\mathbf{5.9\ m/s, 3.7\ m/s.}}$

Example 2: CSI discovers a car at the bottom of a 72 m cliff. How fast was the car going if it landed 22m horizontally from the cliff's edge? (Note that the cliff is flat, i.e. the car came off the cliff horizontally).

Question: $v = ?\ [m/s]$

Given: $h = \Delta y = 72\ m$

$d = \Delta x = 22\ m$

$g = 10.0\ m/s^2$

Equation: $h = v_{iy}t + \frac{1}{2}gt^2$ and $d = v_{ix}t$

Plug n' Chug: Step 1: Calculate the time required for the car to freefall from a height of 72 m.

$h = v_{iy}t + \frac{1}{2}gt^2$ but since $v_{iy} = 0$, the equation simplifies to $h = \frac{1}{2}gt^2$ rearranging for the unknown variable, t, yields

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(72\ m)}{10.0\ m/s^2}} = 3.79\ s$$

Step 2: Solve for initial velocity:

$v_{ix} = \frac{d}{t} = \frac{22\ m}{3.79\ s} = 5.80\ m/s$

Answer: $\boxed{\mathbf{5.80\ m/s}}$

Two-Dimensional and Projectile Motion Problem Set

Draw detailed pictures for each problem (putting in all the data, such as initial velocity, time, etc.), and write down your questions when you get stuck.

1. Determine which of the following is in projectile motion. Remember that "projectile motion" means that gravity is the only means of acceleration for the object after launch.
    a. A jet airplane during takeoff

b. A baseball during a Barry Bonds home run
c. A spacecraft just after all the rockets turn off in Earth orbit
d. A basketball thrown towards a basket
e. A bullet shot out of a gun
f. An inter-continental ballistic missile
g. A package dropped out of an airplane as it ascends upward with constant speed

2. Decide if each of the statements below is True or False. Then, explain your reasoning.

a. At a projectile's highest point, its velocity is zero.
b. At a projectile's highest point, its acceleration is zero.
c. The rate of change of the $x-$position is changing with time along the projectile path.
d. The rate of change of the $y-$position is changing with time along the projectile path.
e. Suppose that after 2 s, an object has traveled 2 m in the horizontal direction. If the object is in projectile motion, it must travel 2 m in the vertical direction as well.
f. Suppose a hunter fires his gun. Suppose as well that as the bullet flies out horizontally and undergoes projectile motion, the shell for the bullet falls directly downward. Then, the shell hits the ground before the bullet.

3. Imagine the path of a soccer ball in projectile motion. Which of the following is true at the highest point in its flight?

a. $v_x = 0, v_y = 0, a_x = 0$, and $a_y = 0$
b. $v_x > 0, v_y = 0, a_x = 0$, and $a_y = 0$
c. $v_x = 0, v_y = 0, a_x = 0$, and $a_y = -9.8\ m/s^2$
d. $v_x > 0, v_y = 0, a_x = 0$, and $a_y = -9.8\ m/s^2$
e. $v_x = 0, v_y > 0, a_x = 0$, and $a_y = -9.8\ m/s^2$

4. A hunter with an air blaster gun is preparing to shoot at a monkey hanging from a tree. He is pointing his gun directly at the monkey. The monkey's got to think quickly! What is the monkey's best chance to avoid being smacked by the rubber ball?

a. The monkey should stay right where he is: the bullet will pass beneath him due to gravity.
b. The monkey should let go when the hunter fires. Since the gun is pointing right at him, he can avoid getting hit by falling to the ground.
c. The monkey should stay right where he is: the bullet will sail above him since its vertical velocity increases by 9.8 m/s every second of flight.
d. The monkey should let go when the hunter fires. He will fall faster than the bullet due to his greater mass, and it will fly over his head.

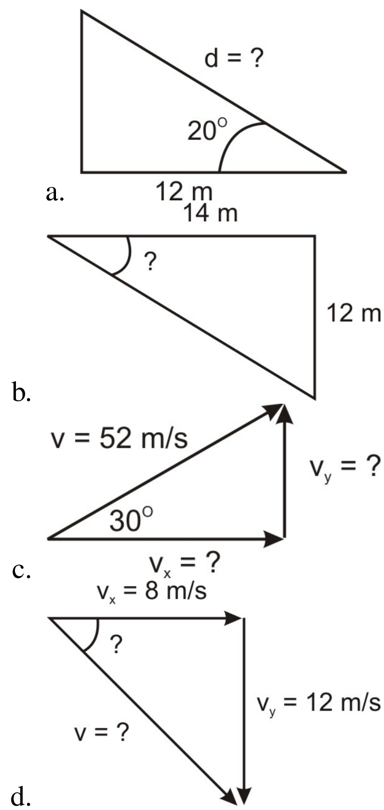
5. You are riding your bike in a straight line with a speed of 10 m/s. You accidentally drop your calculator out of your backpack from a height of 2.0 m above the ground. When it hits the ground, where is the calculator in relation to the position of your backpack?

a. You and your backpack are 6.3 m ahead of the calculator.
b. You and your backpack are directly above the calculator.
c. You and your backpack are 6.3 m behind the calculator.
d. None of the above.

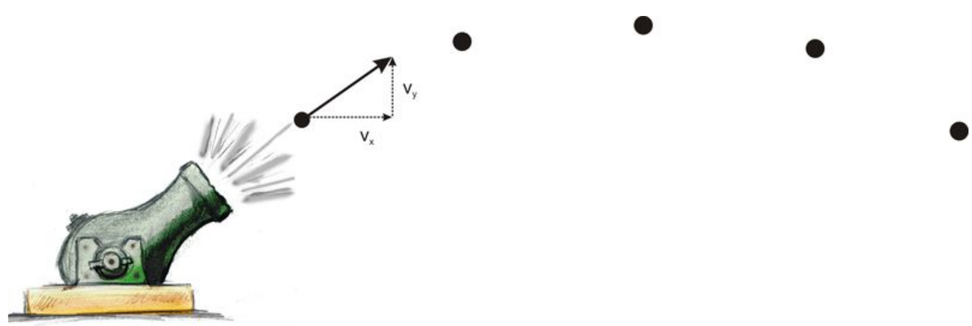
6. A ball of mass m is moving horizontally with speed v_o off a cliff of height h, as shown. How much time does it take the rock to travel from the edge of the cliff to the ground?

a. $\sqrt{hv_o}$
b. $\frac{h}{v_o}$
c. $\frac{hv_o}{g}$
d. $\frac{2h}{g}$
e. $\sqrt{\frac{2h}{g}}$

7. Find the missing legs or angles of the triangles shown.



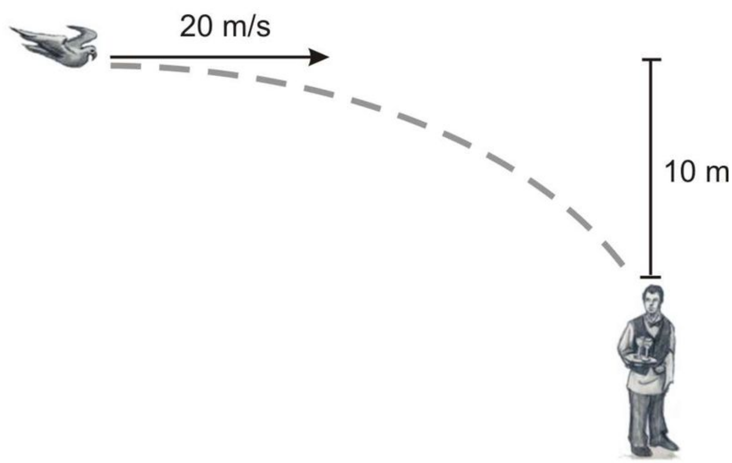
8. Draw in the $x-$ and $y-$velocity components for each dot along the path of the cannonball. The first one is done for you.



9. A stone is thrown horizontally at a speed of 8.0 m/s from the edge of a cliff 80 m in height. How far from the base of the cliff will the stone strike the ground?
10. A toy truck moves off the edge of a table that is 1.25 m high and lands 0.40 m from the base of the table.

    a. How much time passed between the moment the car left the table and the moment it hit the floor?
    b. What was the horizontal velocity of the car when it hit the ground?

11. A hawk in level flight 135 m above the ground drops the fish it caught. If the hawk's horizontal speed is 20.0 m/s, how far ahead of the drop point will the fish land?
12. A pistol is fired horizontally toward a target 120 m away but at the same height. The bullet's velocity is 200 m/s. How long does it take the bullet to get to the target? How far below the target does the bullet hit?
13. A bird, traveling at 20 m/s, wants to hit a waiter 10 m below with his dropping (see image). In order to hit the waiter, the bird must release his dropping some distance before he is directly overhead. What is this distance?



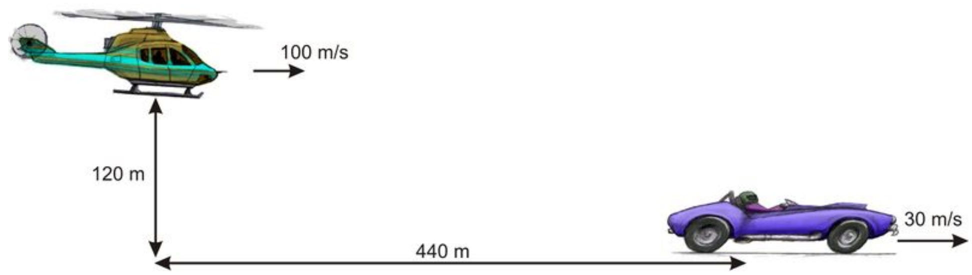
14. Joe Nedney of the *San Francisco 49ers* kicked a field goal with an initial velocity of 20 m/s at an angle of 60°.

    a. How long is the ball in the air? *Hint:* you may assume that the ball lands at same height as it starts at.
    b. What are the range and maximum height of the ball?

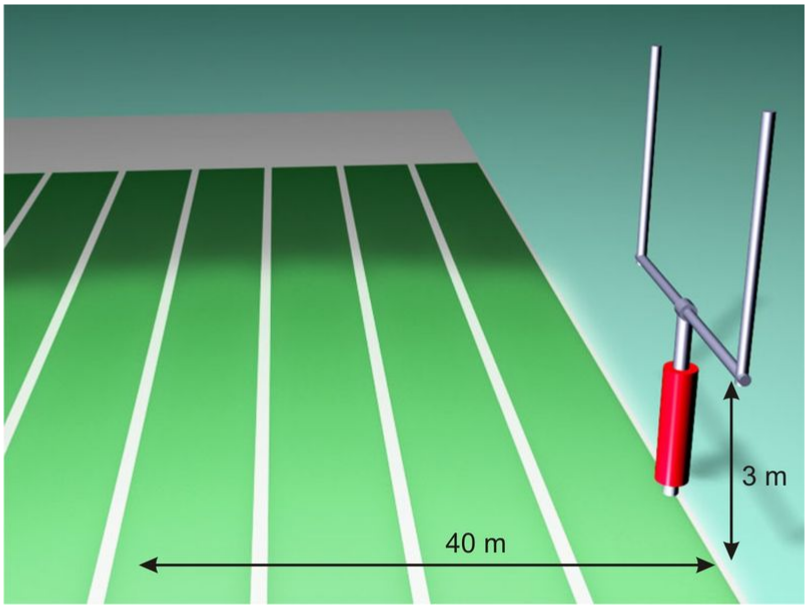
15. Vince Young throws a football. He throws it with an initial velocity of 30 m/s at an angle of 25°. How much time passes until the ball travels 35 m horizontally? What is the height of the ball after 0.5 seconds? (Assume that, when thrown, the ball is 2 m above the ground).
16. How long will it take a bullet fired from a cliff at an initial velocity of 700 m/s, at an angle 30°below the horizontal, to reach the ground 200 m below? (hint: you'll need to use quadratic equation in this one)
17. A diver in Hawaii is jumping off a cliff 45 m high, but she notices that there is an outcropping of rocks 7 m out at the base. So, she must clear a horizontal distance of 7 m during the dive in order to survive. Assuming the diver jumps horizontally, what is his/her minimum push-off speed?

OPTIONAL PROBLEMS

18. Barry Bonds hits a 125 m (450') home run that lands in the stands at an altitude 30 m above its starting altitude. Assuming that the ball left the bat at an angle of 45° from the horizontal, calculate how long the ball was in the air.
19. If Jason Richardson can jump 1.0 m high on Earth, how high can he jump on the moon assuming same initial velocity that he had on Earth (where gravity is $\frac{1}{6}$ that of Earth's gravity)?
20. James Bond is trying to jump from a helicopter into a speeding Corvette to capture the bad guy. The car is going 30.0 m/s and the helicopter is flying completely horizontally at 100 m/s. The helicopter is 120 m above the car and 440 m behind the car. How long must James Bond wait to jump in order to safely make it into the car?



21. A field goal kicker lines up to kick a 44 yard (40 m) field goal. He kicks it with an initial velocity of 22 m/s at an angle of 55°. The field goal posts are 3 meters high.



a. Does he make the field goal?
b. What is the ball's velocity and direction of motion just as it reaches the field goal post (*i.e.*, after it has traveled 40 m in the horizontal direction)?

Answers (using $10\ m/s^2$ **for gravity):**

1-6 Discuss in class

7. (a) 13 m
   (b) 41 degrees
   (c) $v_y = 26\ m/s;\ v_x = 45\ m/s$
   (d) 56 degrees, 14 m/s
8. Discuss in class
9. 32 m
10. (a) 0.5 s
    (b) 0.8 m/s
11. 104 m
12. $t = 0.60\ s, 1.8\ m$ below target
13. 28 m.
14. (a) 3.5 s.
    (b) 35 m; 15 m
15. 1.3 seconds, 7.1 meters
16. 0.5 s
17. 2.3 m/s

18. 4.4 s
19. 6 m
20. 1.4 seconds
21. (a) yes
    (b) 18.6 m/s @ 47 degrees below horizontal